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Experienced teachers talking about their mathematics teaching with linguistically disadvantaged learners

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Abstract
Learners of mathematics who are linguistically disadvantaged for a variety of reasons, including impoverished socioeconomic status, continue to be educationally disadvantaged and at considerable risk of school failure and early dropout. This is the case in many parts of the world. While much has been researched on linguistically disadvantaged learners in the fields of sociology and general pedagogy, little is known about the classroom teaching of mathematical content in language-responsive ways for all learners in school. Experienced mathematics teachers draw on a wealth of knowledge of content teaching in language-responsive ways developed through their practices working with linguistically disadvantaged learners in their classrooms. In this paper we report on interviews with some of these experienced mathematics teachers from seven educational contexts focusing on teaching probability in language-responsive ways. We focus on what we can learn from these teachers that could inform our practice as mathematics teacher educators and our research. We identify three challenges and three practices that add nuance and depth to theoretical research findings and recommendations on language in mathematics teaching, which can potentially develop these findings in more practical and accurate ways.

Keywords Linguistic disadvantage · Classroom practices · Probability teaching · Mathematics and language

Introduction

The crucial role of language for teaching and learning mathematics is widely recognized in mathematics education research. This is reflected in several important surveys such as the International Commission on Mathematical Instruction (ICMI) Study on Mathematics Education and Language Diversity (Barwell et al., 2016), the European Society for Research in Mathematics Education (ERME) (Planas et al., 2018) and the International

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Group for the Psychology of Mathematics Education (PME) (Radford & Barwell, 2016) survey papers on language and mathematics education, as well as three ZDM special issues in the last 5 years (Planas & Schütte, 2018; Erath et al., 2021; Trouche et al., 2023). As this list of surveys indicates, research in mathematics education is at the point of considering the potential reflexive relationships between widely researched insights along with developed theoretical frameworks on the interplay of language for teaching and learning mathematics and everyday practices in schools.

The broader study within which this paper arises focuses on what we can learn from mathematics teachers with a long history of working with linguistically disadvantaged learners in their classrooms. It draws on interviews with these experienced teachers from seven countries, looking at the topic-specific challenges they face and the teaching practices they have developed in response to these challenges. Our use of the term linguistically disadvantaged learners does not imply a deficit perspective on the learners. Our focus is on the topic-specific challenges inherent in the mathematics and the way that mathematics is taught that are often made visible when working with learners who do not share the discourse or language of the classroom in which they are learning, for many different reasons.

Alongside the societal reasons for this study, the scientific reasons are embedded in the problematic construction of knowledge in mathematics teacher education and teaching research *for* teachers that is not informed *by* and constructed *with* them (Goos, 2014; Jaworski, 2003). The reason for an international perspective is to show that teaching mathematics with linguistically disadvantaged learners in local settings is a global challenge. In this paper, we focus on probability for two key reasons: firstly, as researchers we agreed on the scarcity of studies on language in the teaching of probability compared to the other topics in focus in the project; secondly, the teachers we interviewed drew on two of the themes discussed in this paper more extensively or even solely in the interviews focused on probability, namely using contexts and the quantity of vocabulary with specific meanings. While this paper focuses on the topic of probability, the three challenges and three practices have broader implications beyond this topic and resonate with the other interviews collected as part of the broader study. In this paper we answer two research questions: What are the teaching challenges in the topic of probability that mathematics teachers working with linguistically disadvantaged learners experience and that contribute additional perspectives on research-based recommendations? What are the common teaching practices of these teachers that can inform mathematics teacher education research and practice?

The two questions above and the current study also speak about the meta-question of decentering mathematics teacher education. There are research problems and participants whose study is common in mathematics teacher education and hence can be seen as referential or situated in a privileged center. Decentering the domain accounts for the possibility and importance of developing 'other' studies, like the one presented in this paper. Teaching mathematics with linguistically disadvantaged learners is an under-researched problem, and the needs of both these groups of learners and their mathematics teachers tend to be under-represented in the domain. We therefore, address the special issue theme of decentering mathematics teacher education by giving voice to the expertise of teachers whose teaching practices and professional knowledge are of most value for all mathematics learners, particularly learners made disadvantaged because of being linguistically different to mainstream groups.

In the first section, we begin by introducing the broader linguistic basis taken up in mathematics education research and briefly introduce our framework on language and

learning mathematics. Afterward, we review a range of mathematics teacher education research focusing on linguistic aspects of mathematics education and probability specifically before introducing the study and our methodology. We then report on the challenges and practices we identified in response to our research questions before discussing the interplay between them. Finally, implications for mathematics teacher education practice and research are considered that contribute to decentering mathematics teaching experiences and challenges in both socially and pedagogically responsible ways.

Language research informing teaching in mathematics

Research on language in mathematics education has strong roots in working with teachers and learners in language diverse contexts and particularly in working with multilingual learners and learners for whom the language of instruction is not one of their family languages. Early studies revealed that multilingual learners experience limited access to mathematics if teaching is not language-responsive due to institutional obstacles or language barriers (Barwell et al., 2016; Callahan, 2005; Secada, 1992). Years later, Program for International Student Assessment (OECD, 2019) results show that school systems in many countries (still) fail to provide equitable access to mathematics learning for these learners. The observation that monolingual learners can also experience language barriers in mathematics classrooms (e.g., Pimm, 1987; Prediger et al., 2018) points to the complexity of the relationship between language and the teaching and learning of mathematics.

Early research on language in education focused on the construct of academic language proficiency (ALP) introduced by Cummins (1979), who differentiated between CALP (Cognitive Academic Language Proficiency) and BICS (Basic Interpersonal Communicative Skills). This distinction enabled researchers to describe the different language demands of everyday social interactions and those in academic or school contexts (e.g., Erath et al., 2018). Furthermore, studies show that learners' ALP in the language of instruction is the key factor in barriers to mathematics learning (Snow & Uccelli, 2009), not learners' multilingualism (Prediger et al., 2018) although different factors can overlap in different ways for different learners (Planas, 2018). Hence, this linguistic differentiation paved the way for extending language-responsive mathematics teaching from predominantly multilingual learners to all learners who face linguistic challenges, such as those with low ALP in the language of instruction and low previous mathematics achievement and most recently beyond those learners (for an overview see Prediger et al., 2022).

Building on Cummins' differentiation between BICS and CALP, researchers identified three dimensions to characterize the distinction between academic language and everyday language. For example, on the lexical dimension, more complex and unfamiliar words are used; on the syntactical dimension, more complex sentence structures and grammar can be observed; and on the discursive dimension, more demanding discourse practices such as arguing divergent validity claims or explaining meanings occur (e.g., Bailey, 2006; Schleppegrell, 2004; Snow & Uccelli, 2009). Differentiating these three dimensions for understanding and designing mathematics teaching and learning is used widely in mathematics education research (Planas et al., 2018; Erath et al., 2021). A mere focus on vocabulary learning in mathematics classrooms is seen critically, particularly if the offered vocabulary is detached from the mathematical core of teaching (de Araujo & Smith,

2022; Moschkovich, 2002). To what extent and for which learners discursively integrated vocabulary work carefully interwoven with mathematical meaning construction can benefit mathematical learning is still part of ongoing research (see Prediger et al., 2022). However, mathematics education research repeatedly shows the importance of the discursive dimension of academic language for learning mathematics conceptually (Ingram, 2021; Erath et al., 2021; Moschkovich, 2015). In the analysis of our participating teachers' interviews, we describe these language dimensions and varieties in terms of everyday and more formal discourses as this is how our teachers described them.

A sociocultural perspective on language in mathematics teaching and learning (e.g., Planas et al., 2022; Moschkovich, 2015) helps us to make sense of how mathematical concepts and ideas are influenced by the social and cultural contexts in which teachers and learners interact. Drawing on a conceptualization of learning mathematics as "a process of enculturation into mathematical practices, including discursive practices (e.g., ways of explaining, proving, or defining mathematical concepts)" (Barwell, 2014, p. 332), our work focuses on teacher's experiences of these practices with linguistically disadvantaged learners. That is, learning mathematics involves not only mathematical knowledge but also mathematical practices and discourses. Here, discourses comprise "ways of combining and integrating language, actions, interactions, ways of thinking, believing, valuing and using various symbols, tools, and objects to enact a particular sort of socially recognizable identity" (Gee, 2005, p. 21) which Gee refers to as Discourse with a capital D. Thus, two similar ideas are important in our work with teachers: the idea of highlighting the importance of the discursive dimension of language for learning mathematics; and the idea that collective processes of learning mathematics always include establishing mathematical discourses. As the empirical insights below will show, teachers talk about both aspects without explicitly separating them.

In our understanding of language-responsive mathematics teaching, we follow Erath et al. (2021, p. 246) in their focus on "instruction that enhances language for *mathematics learning* […] to mean instruction *through* language, but also *of* language, more precisely of those discourse practices (and lexical and syntactical means for participating in them) necessary for learning mathematics." This includes the functional linguistic idea of seeing lexical and syntactical features of language not as ends in themselves but as means for realizing discourse practices (Schleppegrell, 2007; Snow & Uccelli, 2009). We use the three dimensions within the construct of ALP in our research and our mathematics teacher education practices, considering also the work of researchers emphasizing the importance of valuing and drawing upon learners' rich and diverse previous knowledge (Hunter, 2022).

In what follows, we link to more recent research focusing on implementing research findings into practice by adapting the underlying ideas discussed above for mathematics teacher education and professional development (PD). In doing this, we recognize that much of what follows originates in close observation of mathematics teachers who have developed similar principles and practices through their own experiences that have effectively supported them in teaching linguistically disadvantaged learners. In a recent survey, Erath et al. (2021) systematized six major design principles (left column of Table 1), six categories of teacher moves (right column of Table 1) for enhancing language in mathematics classrooms, as well as teacher practices that are shown to enhance mathematics learning. In our research and teacher education practices, we also share the two overarching orientations formulated by the authors:

- "*Amplify not simplify language* (Pimm, 1987; Schleppegrell, 2007; Walqui & Bunch, 2019; Zwiers et al., 2017) since any approach that reduces language cannot provide the

Table 1 Overview of the Major Design Principles (p. 247) and Categories of Teacher Moves (p. 253) as Identified in Erath et al. (2021)

Major design principles for materials and instruction	Categories of teacher moves for enhancing language
(P1) Engage students in rich discourse practices	(TM 1) Plan and prepare collective discussions that focus on mathematical concepts
(P2) Establish various mathematics language routines	(TM2) Understand and connect students' ideas and mathematics; make them accessible to as many students as possible
(P3) Connect language varieties and multimodal representations	(TM3) Enhance language practices for learning mathematics
(P4) Include students' multilingual resources	(TM4) Encourage student participation in demanding discourse on mathematics
(P5) Use macro-scaffolding to sequence and combine language and mathematics learning opportunities	(TM5) Pay attention to feedback and evaluation of students' mathematics
(P6) Compare language pieces (form, function, etc.) to raise students' language awareness	(TM6) Purposefully use pauses and silence

language learning opportunities required for enhancing the learning of both language and mathematics.

- *Enhance both at the same time, language and mathematics with understanding*, since any approach that addresses only one cannot enhance the language practices involved in learning and doing mathematics (Moschkovich, 2010)" (Erath et al., 2021, p. 247)

We view these orientations as consistent with the theoretical background of this study. Nonetheless, through this study we critically revisit these orientations in ways that reflect the complexity of the classroom contexts within which teachers work by drawing extensively on the experiences of mathematics teachers themselves.

Language in research and development work with mathematics teachers

Extensive research on language in mathematics education has provided a rationale for considering language in mathematics teacher education, and today it informs a range of research on teacher preparation and PD. As a result, mathematics teacher education is becoming increasingly interested in discussing language uses in classroom teaching that support learners' mathematics learning. Following Jaworski's (2003) idea of learning from and with teachers as powerful knowledge for educational research and practice, we review here, research and PD projects based on forms of collaboration between mathematics teachers and researchers who are also teacher educators. These projects are mutually informed by mathematics education research on language and by the teachers' professional insights across learning settings. Some initiatives articulate work with teachers on linguistic and non-linguistic modes of communication in the mathematical materials and teaching practices (e.g., Adler et al., 2022; Neumayer DePiper et al., 2021). Other initiatives focus on linguistic communication in teaching to scaffold student talk and mathematical

discourses to support mathematical learning (Planas et al., 2022; Sztajn et al., 2020). There are also projects paying particular attention to multilingual learners and their mathematics learning (e.g., Kasari & Meaney, 2023; Turner et al., 2019) or to mathematical dialogues in classroom interaction (Sjöblom et al., 2022).

Language challenges in the topic of probability

The topic of probability exemplifies the complexity of the relationship between learning mathematics and language, and most analyses in research have been through the lens of the mathematical language of probability (as summarized in Nilsson, 2009). Mathematics teachers and learners use words such as random, probable, and likely to mean different things (Ingram, 2022; Molnar, 2018), often interacting with everyday uses of these words. There is also a wide range of choices around which (everyday) contexts and which representations to use to illustrate and exemplify probabilistic concepts, properties, and theorems (Eichler & Vogel, 2012; Kvatinsky & Even, 2002; Pfannkuch et al., 2018; Steinbring, 1991). Learners themselves often contextualize probability tasks in different ways (Nilsson, 2009). This range of interpretations and contexts poses particular challenges for mathematics teacher education, as teachers need a coherent understanding of probability, the possible challenges that students may encounter, and an awareness of how the contexts involving probability and randomness can be interpreted differently (Liu & Thompson, 2007).

Learning with and from professional talk with mathematics teachers

Many research and developmental projects including a focus on language have relied on a combination of individual conversations and group meetings with the participant teachers. Hilton and Hilton (2019) interviewed primary school teachers in Australia to investigate their practices, representations and challenges in teaching proportional reasoning, including the use of mathematical language and learners' discussion. In the interviews, different teachers provided different meanings for the concepts of scale, fractions, and relative and multiplicative thinking. This was the basis of further structured discussions in the group meetings. In other cases where individual interviews follow a more conversational style, mathematics teachers have been asked to discuss excerpts or video clips of their teaching and specifically to indicate whether there was something interesting happening and, if so, why it was interesting to them (see Ingram & Coles, 2022; Richards et al., 2021). Although the teachers' talk of their mathematics teaching does not describe teaching as it takes place during instruction, these interviews help to identify what the teachers intend to share with the researcher or the teacher educator and, throughout the conversations, a shared understanding can emerge as to why some aspects of the teaching are selected, emphasized and noted over others. This way of working with teachers is often restricted to a small number of highly committed mathematics teachers in a local context. Yet we also know from mathematics teacher education research that there can be huge variation between educational contexts at the national, local, and classroom levels. In this paper, we focus on the similarities in the challenges and practices experienced teachers describe across diverse educational contexts.

Gorgorió and Planas (2001) carried out interviews with secondary school mathematics teachers in Catalonia—Spain "to identify their understanding of the language problems they face when teaching foreign students … and teaching strategies that could help to overcome

the linguistic barrier" (p. 16). Whereas the language of research in that project was pointing to problems and barriers rather than opportunities and challenges, and the request from the educational administration was strictly concerned with mathematics teaching to "foreign learners", the interviews dismantled the assumption that this was the only group of learners who struggled with the language of instruction in their mathematics learning. The teachers discussed the potentialities of all learners as mathematical contributors in their classrooms, including those in the early process of learning the language of instruction. They included comments about the difficulties in facilitating discussions around unexpected meanings for some mathematical concepts and of unexpected resolutions to mathematical problems evoking real-world contexts. Concurrently Adler (2001) reported interviews with secondary school mathematics teachers in South Africa talking about their mathematics teaching in multilingual classrooms. Three teaching dilemmas, grounded in the data from the teachers, were thematized as: "whether or not to shift toward learned-centered practices that involve more mathematical talk by learners (p. 15), "whether or not to work explicitly on the mathematical language" (p. 15), and "whether or not to switch languages in class" (p. 73). In these two examples, the focus on language in mathematics teaching was primarily related to language diversity as a feature of the learners, and all the teachers' classrooms were in contexts of socioeconomic poverty.

More than two decades later, the knowledge gained about the role of language in all mathematics classroom teaching has widened the possible emphases in the conversations with mathematics teachers for talking about language in mathematics teaching and learning (Morgan et al., 2021). Teachers and researchers can go beyond references to multilingual practices once they have wider views of what language is about. In conversations with mathematics teachers about what seems or does not seem to work and why, concerning their' or others' language practices in mathematics teaching for all learners, they may foresee linguistic challenges in the lexical and discursive dimensions. In Turner et al. (2019), a group of early career mathematics teachers saw language in mathematics teaching as the teaching of technical vocabulary in mathematical discussions following discussions in which language and language diversity were treated as distinct by the teacher educator. In Neumayer DePiper et al. (2021), one of the teachers spoke about the place of language in mathematics teaching: "Often, we as teachers can assume or neglect vocabulary words that students know and do not know…. I neglected to reflect on how important names are and how it makes an impact on understanding the problem" (p. 499). These mentions of language in the lexical dimension point to some recognition of the discursive dimension, in line with the above research (Erath et al., 2021). In Planas et al. (2022), teachers in the two settings of Catalonia—Spain and Malawi introduced aspects in the lexical dimension (e.g., mathematical names within geometry) and the discursive dimension (e.g., explaining the meaning of angle as rotation quantity) in conversations regarding uses of language in secondary school mathematics teaching. All these examples from the literature show a range of contexts in research and PD, including different pedagogical traditions, theoretical lenses and institutional conditions of the teachers and the projects.

Collaborations between mathematics teachers and researchers in research and PD provide access to and interaction with teachers' professional insights and expertise, contributing to comprehensive discussions around the changes in mathematics teaching and learning that changes in language use may entail. Nonetheless, relatively little is still known about the teachers' multiple uses of language in their classrooms as described by them and learned and practiced through their years of teaching mathematics with different groups of learners, some of them linguistically disadvantaged for a variety of reasons. Language-focused conversations with a diversity of mathematics teachers can

help to interpret theoretical research findings on language use in mathematics teaching in more practical and accurate ways. Importantly, the talk of teachers about their or others' mathematics teaching in real classrooms, and their experiences of dilemmas within this, can inform the rich ways we as researchers and mathematics teacher educators make connections between more generally described language practices and mathematical practices to each other. We do this through a focus on specific mathematical topics, as well as in relation to the learning needs of specific groups of learners. Access to and interaction with teachers' talk about language in teaching can thus support the refinement, concretion and improvement of pedagogies and teaching practices that can otherwise remain too theoretical and not always grounded enough in, or attentive to, the data of classroom practice, of mathematical content, and of the learning needs of the learners.

Methods

This small-scale qualitative study involves analysis of fourteen individual interviews with experienced mathematics teachers from seven country contexts, including Catalonia—Spain, England, Germany, India, Malawi, Norway, and South Africa. These teachers are experienced mathematics teachers working with linguistically disadvantaged learners. We wanted to hear from them what they perceived to be the linguistic challenges both they and their learners faced in their mathematics classrooms and the strategies they used to address these challenges. Research tends to focus on teachers' attitudes and beliefs around challenges when implementing new practices, rather than listening to the expertise they might bring. By drawing on the expertise of experienced teachers we are reframing the research practice gap by moving the researchers closer to the teachers rather than trying to move the teachers toward researchers (Goos, 2014).

The teachers were recruited using recommendations from local researchers in each of the contexts and were recognized as experienced mathematics teachers who worked in schools with a range of learners facing linguistic disadvantages in their learning of mathematics, for example, multilingual contexts, where many of the learners were relatively new to the language of instruction, or schools serving areas with high levels of social deprivation where learners may be less familiar or comfortable with academic language. We aimed to recruit teachers from each country context with at least 10 years of mathematics teaching experience in these contexts, but local and cultural differences between the contexts, such as issues with teacher retention or differences in school diversity, meant that a few of the teachers had fewer than ten but more than 7 years of experience. Data collection in seven diverse contexts allows us to forefront a range of teachers' voices; however, the intention is not to highlight differences in a comparison across contexts, but to look at similarities in how experienced teachers are approaching linguistic challenges in their contexts.

All teachers gave informed consent to participate in the study and were given choices about which interviews to participate in, which language to participate in, and the extent to which their own words would be reported. The quotes in this paper are first given in the language spoken by the teacher in the interview and translated by a member of the wider research team where necessary. The study was approved by the ethics committee at the University of the first three authors, and local permission to undertake the research was also sought from each of the jurisdictions in which the research took place.

Data collection

A key aim of the broader project is to examine the expertise that teachers develop through their teaching experiences working with linguistically disadvantaged learners that could enhance and challenge the expertise that mathematics teacher educators and researchers may bring. The semi-structured interviews were designed to identify the challenges experienced teachers faced teaching particular topics to linguistically diverse and linguistically disadvantaged learners and the practices they had developed through their experiences to respond to and support their learners. The focus on specific topics ensured the conversations focused on mathematical aspects of the practices and challenges they described, and also allowed us to consider the variations and similarities within mathematics. The topics were chosen to reflect the diverse linguistic demands within mathematics, including the use of different representations and forms of argumentation and reasoning. These interviews included questions and prompts relevant to the two research questions we focus on in this paper, for example: "Could you give an example of a linguistic challenge that one of your students experienced in a lesson developed to probability and the strategy you used to support the student?"

The interviews focused on four topics within the mathematics curriculum predominantly taught between 11 and 14 years-old in most of the countries: linear equations; proportional reasoning; angle properties; and probability. Not all of the teachers were interviewed about each of the topics, either because they did not teach a particular topic in their current role, or at the request of the teacher. In this paper, we report on the analysis of 14 interviews that focused on probability, where the majority of the teachers explicitly drew upon the distinctions made between everyday and more formal discourses.

Where the teachers chose to be interviewed in English, the second and third authors conducted the interviews. Where another language was chosen, a member of the wider research team who was fluent in the chosen language and English conducted the interviews. All the interviews were audio recorded. Given the multilingual nature of the data and the well-known complexities involved in translations (Geiger & Straesser, 2015), structured analytic notes were constructed in English which summarized the teachers' responses to each prompt, with a similar purpose to transcription of capturing the teacher's perspective faithfully (Ingram & Elliott, 2019; Gibbs, 2018). These analytic notes include direct quotes that seemed pertinent, where a teacher consented to be quoted, but most of these notes were paraphrases and summaries of what was said. During the later stages of the analysis, these notes were revisited in an iterative process to include more direct quotes for the relevant codes reported in this paper as the thematic analysis described next developed.

Analysis of the interviews

The analysis was initially data-driven with codes and themes derived inductively from the data. This reflects the study's primary purpose which focuses on teachers' expertise developed through experience and what we, as mathematics teacher educators and researchers, can learn from this expertise. Coding is seen as a way of "indexing or mapping data, to provide an overview of disparate data that allows the researcher to make sense of them in relation to their research questions" (Elliott, 2018, p. 2851). The coding process described

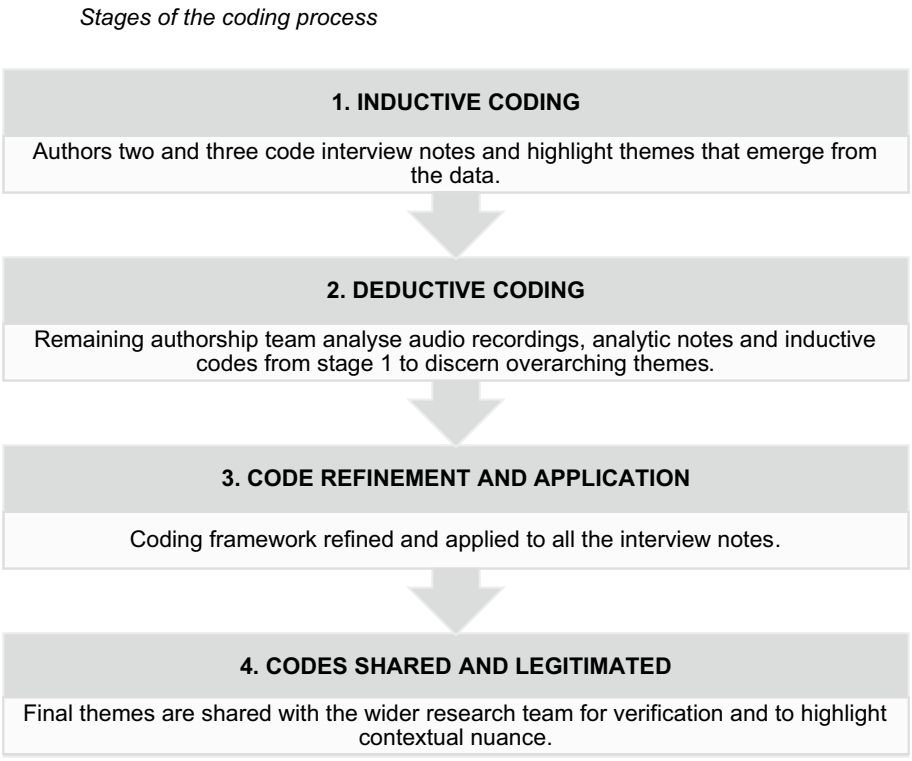

Fig. 1 Stages of the coding process

below and illustrated in Fig. 1 follows a similar format to the hybrid inductive–deductive thematic approach documented by Fereday and Muir-Cochrane (2006).

In the first stage of the coding process the analytic notes relating to the interviews about probability were inductively coded by the second and third authors in NVivo (QSR International Pty Ltd., 2021) according to the linguistic challenges the teachers reported their learners experiencing, and the strategies that the teachers reported using to support their learners. This first stage of coding drew on the language used by the teachers to identify themes across the teachers, without reference to the principles and moves listed in Table 1. As the second and third authors are new to mathematics education research, their backgrounds in mathematics classroom teaching place them closer to the teachers' perspectives, and these authors were intentionally utilized at the inductive stage to ensure a high degree of fidelity to teachers' experiences in the data without a strong lens from previous research. The second stage involved the other three authors revisiting the audio recordings, inductive codes from stage 1, and analytic notes from the interviews conducted in English (the shared language of the authors) in order to refine the code definitions and overarching themes, again with a focus on the language used by the teachers themselves. These authors are all established mathematics teacher educators and researchers, and they brought in knowledge of the field, as well as the design principles and teacher moves from Table 1 to apply a deductive approach to the data and first-stage codes. Themes developed from groups of related codes that arose across the participating countries. The third stage

involved this refined coding framework being applied to the entire collection of analytic notes. The final stage involved all members of the wider research team revisiting the audio recordings for all the interviews about probability to verify and add nuance to the themes and the codes arising from the analytic notes. The wider research team includes ten academic collaborators (in addition to the five authors of this paper), with at least one in each country context.

For this paper, we draw on the themes that address our two specific research questions: What are the teaching challenges in the topic of probability that mathematics teachers working with linguistically disadvantaged learners experience and that contribute additional perspectives on research-based recommendations? What are the common teaching practices of these teachers that can inform mathematics teacher education research and practice? This final stage resulted in a total of six themes which are described and illustrated below in the findings.

Findings

To address our first research question, we focus on three themes related to reported teaching challenges. We then turn our attention to our second research question and the three themes relating to teaching practices. We specifically consider those teaching challenges and practices that can inform both mathematics teacher education practices and research, whether that is by complexifying the application of research recommendations in practice, challenging our assumptions, or by adding nuance to our conclusions. Although we discuss challenges and practices separately, they are deeply connected in the sense that practices often develop in response to challenges and evolve over time. On the other hand, the challenges and practices that the teachers described throughout the interviews indicate the consideration of a diversity of linguistically disadvantaged learners, for whose learning of probability different factors such as multilingualism or ALP in the language of instruction may overlap.

Teaching challenges described by the experienced mathematics teachers

The teachers identified three teaching challenges that are only partially considered in the existing research. These challenges are: (1) everyday language and formal mathematics, (2) finding ways to support learners in explaining their reasoning, and (3) distinctions between similar words and concepts.

(1a) Everyday language and formal mathematics

The first challenge spoken about by several teachers was decisions around using everyday or more formal discourses. For these teachers, the term everyday referred to the words, phrases, and contexts used by learners in their lives outside the mathematics classroom. This was not explicitly contrasted with formal or academic language but instead focused on mathematical features of discourse such as precision. The teachers reported a balance between using formal mathematical discourses and developing learners' understanding of the underlying ideas and processes. One teacher from South Africa talked of this distinction in terms of a goal and the process of reaching this goal with a focus on developing their understanding using everyday or familiar language.

> …but I think that perhaps as an end goal that's not practically attainable from the start and so I think that might be the difference with the mathematics teacher and the mathematician, because I feel we see how that mathematical precision is an end goal, but it's a process to reach and that with linguistically disadvantaged learners, my instinct would not be to strive for that exactness of a definition but to try and get some sort of conceptual understanding going. To allow for the maths to be done, rather than for the maths to be done perfectly or perfectly according to definition. (SA2)

This balance is also reflected in how the teachers talked about vocabulary, signs, symbols, and other representations, and the explanations that accompany the introduction of these. Again, the emphasis was on using everyday language to develop meaning and understanding while recognizing the importance of more formal language. As a teacher from Norway described it, "…we use a lot of the simplest explanations just to be familiar with the signs in the probability" (N2). This development of meaning was also supported by visual representations as explanations and ways of communicating meaning:

> visual almost transcends the words and makes communicating about the visual easier because then you can start talking more in everyday language. You can talk about the coloured-in section of the Venn diagram, or you know the top branch of the tree diagram. Rather than using particular mathematical terminology to know what you're talking about. So I think once you've brought in a visual, it makes communication much easier about what you're trying to do with the probability question. (SA2)

The teachers used various practices to deal with this balance between everyday and formal discourses, which we return to in Sect. 1c. For the teachers in this study, academic language was not described as part of the process of developing understanding but instead as a goal once this understanding had been achieved.

(1b) Ways to support learners in explaining their reasoning

Many teachers talked about the challenge of encouraging and supporting learners to articulate themselves in both verbal and written forms. Many teachers noted how learners often struggle to explain their reasoning in mathematics. For some, this related to learners having restricted access to the language needed, while for others, the challenge related to what counts as a mathematical explanation. For one German teacher, these two challenges are intertwined:

> Ja, schon so die Idee ist da, aber der Sprachschatz ist eben viel zu gering [...] Und sie wollen auch gar nicht in ganzen Sätzen sprechen, weil sie durch die Nutzung des Handys einfach [...] in kurzen knappen Sätzen sprechen oder nicht nicht mal in Sätzen sprechen? Ja und sagen das reicht aus ich sag nein, das ist keine Erklärung, das ist nicht vollständig, also da muss man immer wieder zurückgehen, aber das kann ich nicht nur im Mathematikunterricht leisten. [Yes, so the idea is there, but the vocabulary is far too small [...] And they don't even want to speak in complete sentences, because they simply speak [...] in short concise sentences through the use of the cell phone or don't even speak in sentences? Yes and say that's enough I say no, that's not an explanation, that's not complete, so there you have to go back again and again, but I can't do that only in math classes] (G1)

The teachers shared an assumption of the importance of learners articulating their reasoning and its role in supporting their learning as "it stabilises or becomes more tangible" (N1). This challenge was seen as being particularly acute in probability due to the sheer quantity of new vocabulary, the connections to other mathematics topics such as fractions where learners also may not yet have fully developed ways of articulating their reasoning, and the challenge of explaining ideas within probability. As one teacher from England described it:

> The ability for students to explain probability ideas is a really tricky one that needs quite a bit of support and practice… There has to be quite a bit of discussion about this to get students using the terms the right way and developing their ability to do that (E3)

One of the interviewed teachers in Catalonia—Spain provided a specific example of a learner struggling to explain their mathematical understanding:

> L'any passat li vaig preguntar a una menuda que expliqués per què la probabilitat era un quart per no recordo ben bé quin tret, però era un problema dels de Mendel sobre el color d'ulls dels fills d'una parella. Va assenyalar el número del diagrama dibuixat a la pissarra sense dir res. No era fàcil per ella d'explicar amb paraules el que el quart volia dir matemàticament, crec. [Last year, I asked a learner to explain why the probability of one quarter for I do not remember which feature, but it was a Mendel problem about the eye color of the children of a pair. She pointed to the number in the diagram drawn on the blackboard and said nothing. It was not easy for her to explain with words what was mathematically in the one quarter, I think.] (CS1)

(1c) Distinctions between similar words and concepts

As highlighted in the previous challenge, probability as a topic exemplifies the complexities of acquiring and using topic-specific language. One challenge specifically is distinctions between similar words and concepts within the topic.

> Probability, likelihood, possibility.It is like there are a lot of words that seem like they mean the same but could seem…they sound similar but there are nuances so I think in probability, a lot of the words are…you have to be good at the nuances in the language to understand the words used. (N1)
>
> El vocabulari a probabilitat pot semblar molt proper al vocabulari del dia a dia, i els nanos necessiten aprendre que possible no és probable. Està ple de parelles com aquesta, que hem d'ensenyar. Vull dir ensenyar la diferència entre possible I probable, per exemple, perquè això no és trivial i un repte lingüístic, crec. [The vocabulary in probability can look very close to everyday vocabulary, and learners need to learn that possible is not probable. It is full of many pairs like this, that we need to teach. I mean to teach the distinction between possible and probable, for example, because this is not trivial and a linguistic challenge, I think.] (CS1)

This challenge goes beyond considering the balance between everyday language and formal mathematics but also points to the connections and distinctions within discourses. The language used for talking about probability also connects to discourses from learners' everyday language as well as the language used for talking about other topics within mathematics and other curriculum subjects in school.

Research widely documents the linguistic challenges with talking about probability, as summarized earlier. In the classroom context, the teachers talk of the challenge of working with multiple discourses simultaneously. Teachers need to deal with the combination and concurrent experiences of these challenges, particularly when working with linguistically disadvantaged learners who may not have access to the language used in the teaching and learning of probability.

Teaching practices described by experienced mathematics teachers

Three themes relevant to our second research question focusing on the teaching practices described by the teachers were identified. These are: (2a) Addressing mathematical vocabulary explicitly, (2b) Utilizing topic-specific representations, and (2c) Providing opportunities for listening to and hearing mathematical talk.

(2a) Addressing mathematical vocabulary explicitly

Some teachers mention typically recommended linguistic strategies, such as teaching vocabulary explicitly by defining keywords: "I start by teaching key vocabulary which I think needs to be known in order to actually be able to tackle questions" (SA2). Several teachers also emphasize the importance of teaching the mathematical vocabulary in context, particularly the contexts embedded within classroom tasks and questions.

> [You] need to explore that term in that particular scenario, even then it's quite difficult for students to understand (E1).
> To understand this deep understanding of what a 'sample space' could be or an 'event' when English is your second possibly your third language. You do spend an awfully large amount of time just trying to unpack that word and even students who don't really know it well will then try to ask you to explain it in terms of a context… so you have to spend tons and tons of time really using the language well, in context in the scenario and really not avoiding it. (SA1)

Teachers have nuanced and experience-informed ways of using contexts, and their approaches are often culturally situated with learners' perspectives in mind. One teacher in Malawi (M2) described a particular context used to support learners' understanding of selecting fruits from a basket with and without replacement: selecting fruits without replacement was contextualized by that one wanted to eat those fruits, versus with replacement being fruits that they admired before returning them to the basket. The teacher sees this familiar real-life context as building learners' comprehension of not only the vocabulary, but what the question is asking:

> When I come across such challenges, that's when I resort to use real-life situations because that will make the learners to fully comprehend what the question is saying and what they're supposed to do. (M2)

Some teachers note that a difficulty of bringing their real-life experience into the mathematics is that it often does not match with the real-life experiences and context of their learners. Teachers are cognizant that for a context to be useful in supporting learners' understanding of mathematics, including topic-specific vocabulary, it must be relevant and familiar to the learners.

One teacher described taking the burden of student-centered contexts further, asking their learners to create the connection between themselves and the context: "How do you connect yourself to that question? Is there a scenario where you can be involved and then do that if that makes sense?" (SA1) The teachers found through their experiences that learners bringing themselves into the context of the question allows it to feel more relevant and relatable to learners, ultimately helping them to understand both the vocabulary and the question.

(2b) Utilizing topic-specific representations

Teachers come back to diagrams to approach linguistic challenges in the topic of probability again and again: "If I can ever draw a diagram, I draw a diagram" (E1); "Always ask students to draw the problem" (N3); and "I use thousands of Venn diagrams in the teaching of probability" (CS3). These diagrams are often used to represent the solution space, or possible outcomes, of a probability question.

One teacher points also out how this strategy also supports reading comprehension by making learners slow down and read the whole text carefully. Drawing works "because it forces them to take the text word-by-word…I think that drawing is a very good method that works in that way that I am forcing the students to draw the whole text and to…read every word of the text" (N3). Additionally, the diagram becomes a window into learners' comprehension, as when they make mistakes in their solution, "it's very quick to see which mistake did they do when they did the drawing" (N3). Another teacher in India found that "Visualization helps a lot, when you're actually able to break it down into some kind of diagrammatic representation, so Venn diagrams and tree diagrams are what I use quite a bit." (I3).

This strategy is particularly emphasized in the interviews about probability: "in probability, it's particularly difficult if they don't have that diagram, then actually I would need to guide them and show them a suitable diagram" (E1). Once they have appropriate diagrams and know how to interpret them, the benefits of diagrams seem to transcend the linguistic challenges learners may experience: "If you refer to such a drawing that they are able to see and see what is happening, students do understand" (M3).

Diagrams in probability are not only a way to represent information, but a way to understand the mathematical relationships between the likelihood of different outcomes, and teachers may see them as a convenient way to side-step some of the linguistic challenges inherent in the mathematical vocabulary associated with probability, as illustrated in Sect. 1c.

However, teachers also recognize that the construction and interpretation of probability diagrams are not trivial, and "the literacy of different visuals in this topic is quite broad" (SA2). While diagrams may aid in building some underlying probability understanding or capability, can be difficult to translate back to other forms of explanation without both the mathematical understanding and the linguistic capability. In a similar way to the challenges described in Sect. 1b:

Sometimes they think the drawing itself explains what they think (N3).

> Si els pregunto que expliquin el perquè d'aquella distribució, amb prou feines diuen alguna cosa com que el diagrama ho mostra... Alguns nanos no deixen d'assenyalar el resultat del càlcul or la quantitat de línies del diagrama, I això els val com una explicació. [If I ask them to explain why that distribution, they just say something like because the diagram shows so…Some learners keep on pointing to the result of

the calculation, or to the quantity of lines on the diagram, and this is sufficient for them as an explanation.] (CS1)

The choice of diagram was important for several of the teachers who considered both whether a diagram is to function as an alternative representation to writing or speaking, and whether it needs to be interpretable without words. This requires careful consideration of what the diagram represents mathematically and how learners interpret it, which may vary across different concepts within probability:

Other teachers would say 'If you draw this tree diagram, it does provide the structure for students and they understand why to multiply the fractions', but I'd argue against that, well, no it doesn't, it just shows you the information you've interpreted from the question, it doesn't actually show you why you would multiply some fractions and why you would add some other fractions. (E1)

Els diagrames de Venn ajuden molt amb la probabilittat condicionada i a donar sentit a la regla de la multiplicació. Els dibuixo que se solapin dins d'un rectangle gran, i els dic, d'acord, això és el conjunt sencer de resultats quan feu aquest o aquell altre experiment, i així és com aquest o aquest altre esdeveniment estan relacionats. Puc posar colors al complement d'A, al complement de B... Però els diagrames de Venn no van bé per totes les situacions. No es poden organitzar d'aquesta manera els esdeveniments independents per representar probabilitats. Bé, de fet si que es pot pero no ajuda en res i, a més, és un embolic. Els nanos comencen a pensar que dos cercles sense intersecció proven la independència. [Venn diagrams help much with conditional probability and with making sense of the multiplication rule. I draw them overlapping within a big rectangle, and I tell learners, okay, this is the whole set of all outcomes when you play this or that experiment, and this is how this and this other event relate to each other. I can colour the complement of A, the complement of B… But Venn diagrams do not work for every situation. You cannot organise independent events in this way to represent probabilities. Well, you actually can but it does not help and it is a bit misleading, too. Learners start thinking that two disjoint circles prove independence.] (CS3)

(2c) Providing opportunities for listening to and hearing mathematical talk

An additional way that teachers are conceptualizing opportunities for learners to develop mathematical discourse is through listening to and hearing mathematical talk. The teachers also described strategies to provide these opportunities through supporting discussion and collaborative work with peers to address the challenge described in Sect. 1b.

Some teachers highlight that listening to peers can be helpful for learners' understanding, as "using learners to share their understanding makes it easier for other learners to understand" (M2). This is often seen as a strategy for learners to provide explanations in "ihre Sprache [their language]" (G1), focusing on conceptual understanding without necessarily using the precision of formal mathematics. One teacher notes that it is "easier to learn the words and the concepts with a partner than if I tell them this is what it means and they never use it again," (N1) and, while getting learners speaking and using the words is important, "for students with linguistic challenges…they would have to use words that they might not have" (N1). Listening, however, is somewhat more accessible at first, and teachers describe how peer explanations can be elevated through teacher support.

These teachers are also aware that their own use of mathematical language models mathematical discourse for their learners, and that it is important to use the mathematical vocabulary and to use it precisely. Another way to provide a listening opportunity for learners while interacting with the teacher was offered by one of the teachers from Catalonia—Spain, where learners prompt the teacher in a sort of "inverse game":

> Els pregunto que comencin una frase matemàtica que jo hauré de completar, i ells hauran de dir si està ben acabada. Això funciona amb tots els alumnes, i amb tots els temes. Fins i tot pels alumnes que mai hi juguen, almenys directament, parlant. Sempre penso, estan a classe, escoltant les frases i aprenent mentre escolten.
> [I ask them to start a mathematical sentence that I will need to complete, and they will need to say if it is completed well. This is something that works with all learners, and all topics. Even for those learners who never play the game, at least directly by talking, I always think, they are in the classroom, listening to the sentences and learning from this listening]. (CS2)

Learning from listening and hearing is not purely experiencing the use of mathematical vocabulary. Part of mathematical discourse is internal in the questions we might ask ourselves or the mental explanations we develop. As a teacher in Norway states, as learners "you don't have your inner monologue yet, you need someone else to tell you and then you can internalise it" (N1).

These teaching experiences and practices described respond to the challenges these teachers identify through their experiences working with linguistically disadvantaged learners. They also illustrate the complexity of the decisions these teachers make to support these learners, offering more nuance to research recommendations around practices such as making connections to learners' personal experiences.

Discussion

Teachers face many linguistic challenges in their teaching, but through years of experience they have developed a range of practices to address these challenges. While many of the practices arising from this study agree with those shown in previous literature, our study provides crucial confirmation from teachers working in diverse contexts. However, two distinct contributions arise from the findings of this study, adding to our understanding of practices that support linguistically disadvantaged learners in mathematics classrooms. In this section we discuss these contributions in turn. The first considers teachers' holistic approaches to vocabulary, both in intertwining the lexical and discursive dimensions of language, as well as distinguishing the relationships between complex mathematical vocabulary. The second contribution focuses on the importance of learners listening to and hearing mathematical discourses of both peers and teachers and how listening may contribute to learning mathematics.

Teachers' approaches to vocabulary

The findings above confirm that teachers face challenges regarding the balance between everyday language and formal mathematics. Interestingly, teachers in our data do not focus on formal mathematical language, rather they speak of prioritizing mathematical understanding, and therefore, using everyday language over formal mathematics and

precision, despite consensus in the literature that mathematical language should be amplified rather than simplified (Erath et al., 2021; de Araujo & Smith, 2022; Walqui & Bunch, 2019). This was both in relation to topic-specific vocabulary and in relation to what counts as a mathematical explanation specifically in probability. Particularly interesting is the finding that teachers do not talk about academic language which is given particular attention in many studies on language-responsive teaching (Erath et al., 2021).

The complexity of topic-specific language not only challenges learners' opportunities to interpret the tasks, contexts and problems they encounter, but it also challenges their opportunities to make sense of the underlying ideas and proves challenging when learners try to explain their own reasoning. Most teachers' reported practices fit within a perspective that sees mathematical discourse developing from everyday discourse, but not as a means for allowing learners to "gain control over the mathematical concepts as their understanding develops" (Barwell, 2016, p. 343). Instead, mathematical language was seen as developing after understanding of the ideas within probability rather than in an intertwined way as suggested in recent research (Erath et al., 2021).

A firm grasp of specific mathematical vocabulary and its underlying concepts, encompassing its form and function, is an important component of mathematical proficiency in not only probability. There are also established links between understanding mathematical vocabulary and mathematics performance for both primary and secondary learners (Peng & Lin, 2019; Powell et al., 2020; Ünal et al., 2021). However, there is limited research around the use and understanding of vocabulary within topics such as probability and how to approach this within the classroom. Many of the teachers interviewed recalled their difficulty with differentiating between terms and concepts within probability and how their learners struggled to work within this obscurity.

Thompson and Rubenstein (2000) originally outlined several vocabulary-related challenges that teachers may encounter in the classroom, with one of these issues being that "some mathematical words are related, but learners confuse their distinct meanings" (p. 569), which is also evident in the challenges identified by the teachers in this study. However, Thompson and Rubenstein offer no specific practices to tackle this challenge, only providing an overview of general practices that teachers could use to tackle issues relating to mathematical vocabulary, including attaching vocabulary to established ideas. The limited research about how teachers can tackle topic-specific challenges of differentiating between related but distinct vocabulary highlights an area in need of further exploration and development within the language domain of mathematics education.

The teachers in this study also spoke about contextualizing mathematical vocabulary using culturally situated scenarios that are relevant and familiar to their learners. This practice is supported in other research focusing specifically on vocabulary learning (e.g., Lawrence et al., 2017). In the presented study, teachers' practices went beyond learning the vocabulary to also their learners' understanding of the topic-related ideas. This is consistent with researchers who have argued that vocabulary should be taught in connection to mathematical concepts and their use (Moschkovich, 2002; Prediger et al., 2022) and should be situated in rich discourse practices (Ingram, 2021; Erath et al., 2021; Moschkovich, 2015). Our teachers also place particular emphasis on focusing specifically on rich, learner-relevant contexts. Learner-relevant contexts are necessarily culturally situated, valuing, and drawing upon learners' existing 'funds of knowledge' (Hunter, 2022). Our teachers note that these contexts not only aided in comprehension, but also led to the responsive use of learners' multilingual resources to build fluency with mathematical vocabulary in the language of instruction. Adler

(2001) recommends acknowledging the perspectives that learners share in the classroom, and our teachers drew on these perspectives in the contexts they used.

The role of student listening and hearing

Most teachers noted that eliciting learner talk within the topic of probability is a challenge, as learners struggle to explain their mathematical reasoning in the classroom or do not see a need for further explanation. They all recognized the importance of "encourage[ing] student participation in demanding discourse on mathematics" (Erath et al., 2021, p. 253), though few offered explanations as to why they thought this was important. A key challenge highlighted by many teachers was that learners struggled to articulate their thoughts or mathematical reasoning. To tackle this, teachers highlighted the importance of learners hearing mathematics and the creation of opportunities where learners can listen to both their peers and the teacher.

Moschkovich (2021) and Xu and Clarke (2019) note that it is important to consider both silence and talk when referring to participation in the classroom. Moschkovich (2021) further notes various factors that could explain learners' challenges in articulating their reasoning, including recent experiences and lack of confidence, some of which could be associated with linguistic disadvantage. Many of our teachers did mention that the quantity and complexity of the vocabulary involved in probability was an aspect of the topic that many learners struggled with, making this articulation of thinking and reasoning more challenging. Teachers' practices to overcome these challenges mainly consisted of modeling explanations and allowing the learners to work with their peers, but there were also instances of incorporating probing questions, class discussions, and sentence completion.

The practices of facilitating peer discussion to strengthen learners' understanding and reasoning is also supported by research (Moschkovich, 1996, 1999). However, MacGregor (2002) found that learners working in groups may also struggle with expressing themselves clearly and understanding their peers. Additionally, other researchers argue that "interacting with peers alone will not lead to the development of the mathematics register" (Schleppegrell, 2007, p. 148), which may, in turn, lead to a diminished conceptual understanding which is seen to partially develop from the use of appropriate mathematical terminology (MacGregor, 2002; Schleppegrell, 2007; Veel, 1999). Teacher moves that support learners in becoming active in mathematical discourses are summarized in Table 1. However, our teachers described using these moves but were still challenged by learners' difficulties, or silence, in articulating their reasoning.

To address these limitations, our teachers also talked about the importance of learners listening to and hearing mathematical discourse that could arise from teachers modeling this discourse in situations where this is challenging for some learners. Several studies consider teachers' listening (Davis, 1997; Doerr, 2006) and its role in how teachers respond to learners' contributions (Lim et al., 2020); however, less attention is paid in mathematics education research to the functions of learners' listening, though it underpins arguments around considering silence as a form of participation (Moschkovich, 2021; Xu & Clarke, 2019).

Conclusion

This study examined experienced teachers' perspectives on the teaching and learning of probability with linguistically disadvantaged learners, specifically mathematics teachers whose teaching develops in contexts of poverty and of diverse types of marginalization. Teachers were recruited from a wide variety of contexts, thereby adding nuance to and extending existing research and mathematics teacher education practices in this area. By talking with teachers about their mathematics teaching with linguistically disadvantaged learners, and discussing findings arising from what they told us, this study contributes to building the domain of mathematics teacher education as a site more attentive to teachers and learners whose needs and strengths are often absent from mainstream research.

The findings illustrate the complexity and topic-specificity around decisions teachers need to make in response to the challenges they face. They also highlight the limitations and issues of applicability of recommendations arising from research. More specifically, this study contributes two main findings. The first is that teachers find that their learners struggle to differentiate between related but distinct terminology within the topic of probability, which is circumnavigated through culturally relevant and learner-centered contextualization. The second contribution highlights the importance of learner listening in the development of mathematical understanding and how teachers provide the sites for this to occur. Providing opportunities for listening to and hearing mathematics is again full of challenges working with learners who are linguistically disadvantaged and who struggle with seeing themselves as participants, not only in the mathematical discourse and talk, but also in the classroom, in the school, and even in society.

As mathematics teacher educators striving to develop a reflexive relationship with teachers working in the complexity of the everyday classroom, it becomes more important to listen to the expertise experienced teachers have developed when working with linguistically disadvantaged learners in their classrooms. This is fundamental to recentering mathematics teacher education, that is, to move the domain to approach the various phenomena through a multiplicity of linguistic, social, and cultural scenarios inherent in being a teacher and a learner of school mathematics. By including the perspectives of teachers from a variety of contexts, and especially those working with historically marginalized learners, existing research can be amplified through a more nuanced lens, thereby making research applicable to a wider range of situations and contexts.

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