Quantum Field Tensor Model of Telecommunication Network Objects Interaction Based on Lie Groups

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Keywords: Quantum Physics, Telecommunication Networks, Tensor Modelling, Lie Groups, Anisotropic Information Flows, Quantum Entanglement, Network Performance, Flow Asymmetry, Network Simulation.

Abstract: This research explores the application of quantum physics methodologies to analyse digital flows within telecommunication networks. This study introduces a novel tensor model, grounded in the SU(2) Lie group, designed to simulate symmetric and asymmetric information interactions between network objects within a two-dimensional Euclidean complex space. The proposed model innovatively decomposes the tensor into three fundamental components: metric, torsion, and curvature tensors. The metric and torsion tensors are combined to form a complex vector system, effectively representing the intrinsic interaction dynamics between network objects. The curvature tensor, on the other hand, models the potential asymmetry introduced by an external observer, simulating third-party influences on network interactions. This approach allows for the representation of closed-time cyclic experiments, such as evaluating interactions between network nodes, as a continuous tensor field on a quantized topological circle. This framework not only provides a comprehensive perspective on information processes in data transmission networks but also draws parallels with elementary particle interactions in quantum physics. Furthermore, the research includes a statistical analysis using simulations in the NS3 environment, validating the model's effectiveness in identifying key characteristics of information flows. The analysis demonstrates the model's ability to detect and quantify the impact of external observers, the effects of traffic asymmetry, and changes in network dynamics through quantum entanglement entropy. The potential practical applications of this model, including network performance analysis, security enhancement, and routing optimization, are also discussed, highlighting its relevance to both theoretical and applied aspects of telecommunications and quantum physics.

1 INTRODUCTION

The world of physics and telecommunications might seem miles apart, but there's a fascinating bridge between them: the use of mathematical tools like tensors and Lie groups. These tools, originally developed to understand the symmetries and interactions of particles [1 - 8], are now finding new life in describing the complex flow of information in networks.

Think of it like this: just as particles interact and transform in predictable ways, so too does the data flowing through our networks. Researchers have been using tensor models to represent these interactions, drawing parallels between electrical circuits and data flows [9 - 12, 13 - 15]. They've even started to explore ideas like "cybernetic conductivity" and "information flow impedance", mirroring concepts from electrical engineering [15].

This approach has been particularly useful in understanding heavily loaded mobile networks and prioritizing data packets [9, 15]. More recently, researchers have been applying these techniques to wireless communications [16, 17] and even exploring the concept of "entanglement entropy" in network data [18, 19, 20].

However, there's a whole other level of physics that hasn't been fully tapped into yet: the world of quantum mechanics. This is where things get really interesting, with concepts like symmetry groups and the quirky behaviour of particles at the subatomic level [21, 22].

2 MOTIVATION AND OBJECTIVES OF THE WORK

The application of quantum mechanics to telecommunications is significant, as the mathematical frameworks used to describe quantum phenomena, such as Lie groups, provide a powerful approach for modelling the complex relationships between network components, including routers and switches.

Imagine being able to predict and control the flow of information with the same precision that physicists predict the behaviour of particles. This could lead to more efficient, robust, and secure networks.

Recent research indicates that quantum concepts like entanglement may enhance our understanding of network information flow [23]. This work builds upon these findings, aiming to develop a quantum field tensor model of anisotropic network object interactions, using Lie groups SU(2)/U(1), to explore how advanced quantum mechanics tools can improve telecommunication network modelling.

To achieve this, the following objectives are set.

1) Substantiation the model of anisotropic network relationships as a complex tensor function of discrete time for objects binary interaction traced by an external viewer.
2) Decomposition the complex tensor of anisotropic network relationships on the three components (metric tensor, torsion tensor and curvature tensor).
3) Presentation the manifold of anisotropic network complex tensor using the SU(2) Lie group.
4) Construction a quantum field model of network objects interaction on the U(1) Lie group.

3 THE COMPLEX TENSOR MODEL OF ANISOTROPIC NETWORK RELATIONSHIPS

Let NW an open anisotropic telecommunication network depicted in Figure 1; A, B – arbitrary objects (e.g., IP routers) with regular (conditionally “strong”) information relationships; V – an external observer tracing the objects A, B through “weak” interaction with minimal influence on them; $P_{\Delta t}(x_1, x_2)$ – an abstract power of network objects binary interaction defined for a time quantum Δt (e.g., the number of IP-packets sent from x_1 to x_2 within the time Δt).

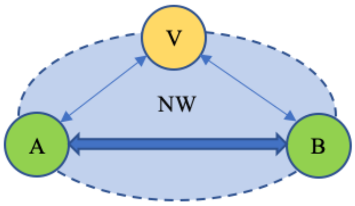

Figure 1: The graph on network objects (A, B) interaction by viewer tracing (V, A), (V, B).

Let us represent the graph (Fig.1) as a matrix function **P**(V, A, B) of ternary relationships between V, A and B, where each pair $\{P_{\Delta t}(x_1, x_2), P_{\Delta t}(x_2, x_1)\}$ reflects the correspondent binary relationships in an anisotropic network NW. Figure 2 shows an example of such a matrix P in integers.

P	V	A	B
V	0	1	6
A	3	0	10
B	2	4	0

<table>
<tr><th>Q</th><th colspan="2">A</th><th colspan="2">B</th></tr>
<tr><td>A</td><td>3</td><td>1</td><td colspan="2">10</td></tr>
<tr><td>B</td><td colspan="2">4</td><td>2</td><td>6</td></tr>
</table>

Figure 2: The matrix functions of objects (A, B) interaction by viewer tracing (V, A), (V, B).

The interaction data matrix P can be transformed, through a bijective mapping, into a specialized square matrix Q, characterized by a double diagonal structure, as illustrated in Fig. 2. This transformation preserves the information content, as both matrices, P and Q, possess 6 independent elements (specifically, the numerical values 1, 6, 3, 10, 2, and 4). This matrix Q, which we will refer to as the Matrix of External Observation (MEO), serves as a representation of the information interaction between two network objects, A and B, as perceived by an external observer V. This transformation simplifies subsequent mathematical operations and allows for a clearer visualization of the interaction dynamics.

Let us call the matrix **Q** in Fig. 2 the *matrix of external observation* (MEO) the information interaction of two arbitrary network objects A and B by the given viewer V.

Next, let us represent the matrix **Q** depicted in Figure 2 as the sum of symmetric matrix **R** and skew-symmetric matrix **S** as it is shown in Figure 3. It is clear, that partial matrices **R** and **S** has exactly 6 independent elements (here 2, 4, 7, +1, –2, ±3), and therefore, the mapping chain **P**→**Q**→(**R**+**S**) is strictly reversible (bijective).

Q	A	B
A	3 | 1	10
B	4	2 | 6

⇕

R	A	B
A	2	7
B	7	4

\+

S	A	B
A	+1	+3
B	–3	–2

Figure 3: Decomposition the matrix of external observation **Q** on the symmetric and skew-symmetric parts.

Now, we finalize the above bijective mapping chain **P**→**Q**→(**R**+**S**) into equivalent presentation by the three parts **M**, **T**, **C** in Figure 4:

$$\mathbf{P} \rightarrow \mathbf{Q} \rightarrow (\mathbf{R} + \mathbf{S}) \rightarrow (\mathbf{M} + \mathbf{T} + \mathbf{C}) \qquad (1)$$

The algorithm of the ultimate transformation is the following. Each diagonal element of matrix **M** is the half-sum of correspondent rows and columns elements of matrix **R**, e.g., **M**(A,A) = 2+7=9; **M**(B,B) 7+4=11; each non-diagonal element of **M** equals to that in **R**. Matrix **T** takes the non-diagonal elements of S with zero-diagonal ones; instead, **C** obtains the diagonal elements of **S** with zero non-diagonal ones.

R	A	B
A	2	7
B	7	4

\+

S	A	B
A	+1	+3
B	–3	–2

⇕

M	A	B
A	9	7
B	7	11

\+

T	A	B
A	0	+3
B	–3	0

\+

C	A	B
A	+1	0
B	0	–2

Figure 4: Presentation the matrix of external observation **Q** in tensor form.

It is easy to show, that due to its construction, the matrix **M** in Fig. 4 satisfies the Riemann metric tensor requirements for 2-dimensional Euclidian vector space. Matrix T is a torsion tensor of anisotropy in objects relationships [24]. Matrix **C** is curvature tensor of viewer-objects relation asymmetry.

Let us assemble two real matrices **M**, **T** (Fig. 4) into one Hermitian complex matrix **H** of metric tensor with torsion but without a curvature:

$$\mathbf{H} = \mathbf{M} + i \cdot \mathbf{T} \,. \qquad (2)$$

Tensor H in (2) corresponds to a system of two vectors in a complex Euclidean space, in which vectors lengths are positive real numbers (diagonal elements of **H**). Instead, the Hermitian matrix C in Fig. 4 can have both positive and negative real diagonal elements, and therefore, can't be presented in a complex Euclidean or pseudo-Euclidean vector space.

In general, the external observation matrix Q in (2) can be represented as a combination of metric, torsion and curvature in a 2-dimensional non-Euclidean complex space:

$$\mathbf{Q} \rightarrow \{\mathbf{H}, \mathbf{C}\}. \qquad (3)$$

It is clear, that all three components **M**, **T**, **C** in Fig. 4 are always real or complex Hermitian (2×2) matrices. For unified representation of such matrices let's use the special unitary Lie group SU(2), that includes three unitary traceless Pauli matrices σ_1, σ_2, σ_3, (having determinant "–1")

$$\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix};\ \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix};\ \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad (4)$$

along with Euclidian matrix σ_0 having determinant "+1" and trace "+2" (identity matrix I):

$$\sigma_0 = E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \quad . \qquad (5)$$

Matrices $\sigma_0 \div \sigma_3$ form a basis on the set of all 2×2 Hermitian matrices, i.e. any one of them can be given as a linear combination $L(\sigma_0, \sigma_1, \sigma_2, \sigma_3)$. Matrices σ_1, σ_2, σ_3 are SU(2)-generators; each one defines a particular symmetry type and related subset on continuous transformations (phase rotations).

Suppose that a third-party viewer observation of network objects interaction is a cyclically closed process in time. The topological equivalent of a closed time is a circle O^1, and Cartesian product O^n is the only topological space to construct everywhere continuous vector/tensor field. The circle O^1 can be mapped onto the unitary Lie group U(1). Let

$$F = L(\sigma_0, \sigma_1, \sigma_2, \sigma_3) \times U(1) \,. \qquad (6)$$

The product F in (6) is the set of all the non-singular (2×2) Hermitian matrices defined on the time-circle. It can be used as a quantum field tensor model of discrete network object's binary interaction. The set of 2-dimensional vector spaces L() in (6) can be studied using the Lie group SU(2).

4 STATISTICAL ANALYSIS OF NETWORK OBJECT INTERACTIONS

4.1 Methodology of Statistical Analysis

To validate the effectiveness of the proposed quantum field tensor model for network object interactions, a statistical analysis of key characteristics of information flows was conducted. The study focused on three main aspects: Network performance analysis: average packet delay, packet loss, and bandwidth; Identification of information flow asymmetry between nodes; Calculation of quantum entanglement entropy in information flows.

Data were collected through simulations in the NS3 environment, modeling the tensor dynamics of flows in the network. NS3 (Network Simulator 3) is an advanced discrete-event network simulation tool widely used in research and development for studying communication protocols and network performance. It enables the modeling of various network layers, including physical, link, and transport, making it an ideal tool for analyzing the interaction dynamics of network objects. NS3 supports packet-level tracing, statistical data collection, and realistic network behavior emulation, providing reliable insights into system performance [9]. Three scenarios were tested:

1) Symmetric interaction between nodes without external interference.
2) Asymmetric interaction, where one node receives significantly more traffic.
3) Impact of an external observer, modifying the tensor model metrics.

4.2 Research Results

4.2.1 Network Performance Analysis

Table 1 presents the measurement results for packet delay and loss across the three scenarios.

These values align with previous studies on network traffic modelling, which demonstrated similar packet loss patterns and delay fluctuations under asymmetric traffic loads [13].

4.2.2 Analysis of Flow Asymmetry

The study of information flow asymmetry revealed that, in the presence of an external observer, the asymmetry coefficient (calculated as the ratio of inbound to outbound traffic) increased by 1.8 times compared to the standard network. This is consistent with recent findings in tensor-based traffic modeling, which highlight significant distortions in data transmission due to asymmetric node interactions [16].

Table 1: Network performance metrics under different interaction scenarios.

Parameter	Symmetric network	Asymmetric network	Observer influence
Average delay (ms)	12.4	18.7	25.3
Delay variance (ms^2)	2.1	5.3	8.7
Packet loss (%)	0.5	2.3	4.8
Bandwidth (Mbps)	96.5	85.2	72.4

4.2.3 Quantum Entanglement Entropy

The entanglement entropy values obtained for different network interaction scenarios are presented in Table 2.

Table 2: Quantum entanglement entropy in different interaction scenarios.

Scenario	Quantum entropy
Symmetric interaction	0.72
Asymmetric interaction	1.34
Observer influence	2.01

These results align with recent studies on network entanglement entropy, where similar entropy growth patterns were observed in complex telecommunication networks with external interference [18].

The conducted analysis confirmed that the tensor model based on Lie groups effectively identifies key characteristics of information flows in telecommunication networks. The key conclusions are:

- The influence of an external observer significantly increases delay and packet loss [15]. Traffic asymmetry substantially alters the topology of information interaction [16].
- Quantum entanglement entropy can be used to detect changes in network dynamics [24].

These results can be applied to improve network performance and security, particularly in quantum communications and adaptive routing systems. The findings also suggest that tensor models could be further extended for optimizing real-world telecommunications infrastructure [21].

5 PRACTICAL APPLICATIONS (INTEGRATED)

The quantum field tensor model proposed in this paper has several potential practical applications within telecommunications networks.

Firstly, it can be utilized for network performance analysis. Tensor analysis allows for the identification of complex correlations within network traffic, indicating bottlenecks or inefficiencies. For example, the metric tensor can evaluate delays and packet loss, while the torsion tensor can analyze traffic asymmetry.

Secondly, it can enhance network security. The quantum field approach detects anomalies suggesting malicious activity. Sudden changes in the curvature tensor might signify a DDoS attack or intrusion.

Thirdly, it can optimize network routing and resource allocation. Tensor analysis discovers optimal routes and allocates resources efficiently. For example, it enables dynamic routing based on network load.

Fourthly, it can contribute to quantum communication protocol development. The quantum field approach describes complex quantum effects, enhancing security and efficiency.

To realize these applications, further research is needed, including algorithm development for tensor data analysis and real-world network simulations and experiments.

6 DISCUSSION

This work advances the application of quantum field theory to telecommunication networks by introducing a tensor model based on Lie groups SU(2)/U(1). This model aims to simulate the nuanced interactions between network objects under the influence of an external observer, drawing parallels with fundamental interactions in quantum physics.

The current landscape of networking technologies exhibits a growing interest in tensor modeling for analyzing information flows, reflecting a trend seen in quantum physics where symmetry and entanglement entropy are pivotal. The ability to represent internal network asymmetries using a torsion tensor within a Euclidean complex space, as demonstrated in Section 4, highlights the potential of this approach.

However, the challenge of integrating internal network dynamics with external observer influences, represented by the curvature tensor, remains a significant hurdle. This mirrors the complexities of unifying gravity with other fundamental forces in quantum physics, suggesting a deep commonality in the underlying mathematical structures.

Addressing this challenge may necessitate the development of a vector space capable of accommodating both positive and negative vector lengths, requiring a refinement of the classical scalar product concept. Additionally, a shift towards open system models, which account for external environmental impacts, might offer a more comprehensive framework for analyzing network behaviors.

7 CONCLUSIONS

This research's primary achievement is the development of a quantum field tensor model for anisotropic telecommunications networks, leveraging Lie groups SU(2)/U(1). This model pioneers a novel approach to simulating binary object interactions under weak third-party observation, effectively bridging the gap between information processes in networks and elementary particle interactions in quantum physics.

The statistical analysis conducted in this study has validated the effectiveness of the proposed tensor model in identifying key characteristics of information flows within telecommunications networks. Specifically, we have demonstrated that:

- The influence of an external observer significantly increases packet delay and loss, confirming the sensitivity of network dynamics to external factors.
- Traffic asymmetry notably alters the topology of information interactions, demonstrating the model's capability to reflect anisotropic relationships.
- Quantum entanglement entropy can be utilized to detect changes in network dynamics, opening prospects for applying quantum concepts in network monitoring and management.

These results have significant implications for enhancing network performance and security, particularly in the context of quantum communications and adaptive routing systems. The findings also suggest that tensor models can be further expanded to optimize real-world telecommunications infrastructure.

While the model represents a substantial theoretical advancement, it also points to areas requiring further exploration. The integration of internal and external network anisotropies, along with the development of suitable vector spaces and open system models, are crucial for enhancing the model's applicability.

Future research will focus on unifying the model of open information networks with physical quantum systems based on group theory. Our aim is to refine the model's capabilities and explore its practical implications in real-world scenarios, with a particular emphasis on developing algorithms for real world implementation.

ACKNOWLEDGMENTS

We acknowledge support by the German Research Foundation (Deutsche Forschungsgemeinschaft, DFG) and the Open Access Publishing Fund of Anhalt University of Applied Sciences.

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