Received: 19 October 2023 | Accepted: 16 August 2024

DOI: 10.1111/bjet.13517

ORIGINAL ARTICLE

British Journal of Educational Technology BERA

Basic mathematical skills and fraction understanding predict percentage understanding: Evidence from an intelligent tutoring system

Markus Wolfgang Hermann Spitzer[1] | **Miguel Ruiz-Garcia**[2,3] | **Korbinian Moeller**[4,5,6]

[1]Department of Psychology, Martin-Luther University Halle-Wittenberg, Halle, Germany

[2]Departamento de Estructura de la Materia, Física Termica y Electronica, Universidad Complutense de Madrid, Madrid, Spain

[3]Grupo Interdisciplinar de Sistemas Complejos (GISC), Universidad Carlos III de Madrid, Madrid, Spain

[4]Center for Mathematical Cognition, School of Science, Loughborough University, Loughborough, UK

[5]Leibniz-Institut fuer Wissensmedien, Tuebingen, Germany

[6]LEAD Graduate School and Research Network, University of Tuebingen, Tuebingen, Germany

Correspondence
Markus Wolfgang Hermann Spitzer, Department of Psychology, Martin-Luther University Halle-Wittenberg, Emil-Abderhalden-Straße 9, 06108 Halle(Saale), Germany.
Email: markus.spitzer@psych.uni-halle.de

Research on fostering learning about percentages within intelligent tutoring systems (ITSs) is limited. Additionally, there is a lack of data-driven approaches for improving the design of ITS to facilitate learning about percentages. To address these gaps, we first investigated whether students' understanding of basic mathematical skills (eg, arithmetic, measurement units and geometry) and fractions within an ITS predicts their understanding of percentages. We then applied a psychological network analysis to evaluate interdependencies within the data on 44 subtopics of basic mathematical concepts, fractions and percentages. We leveraged a large-scale dataset consisting of 2798 students using the ITS *bettermarks* and working on approximately 4.1 million mathematical problems. We found that advanced arithmetic, measurement units, geometry and fraction understanding significantly predicted percentage understanding. Closer inspection indicated that percentage understanding was best predicted by problems sharing similar features, such as fraction word problems and fraction/natural number multiplication/division problems. Our findings suggest that practitioners and software developers may consider revising specific subtopics which share features with percentage problems for students struggling with percentages. More broadly, our study demonstrates how evaluating interdependencies between subtopics covered within an ITS as a data-driven approach can provide practical insights for improving the design of ITSs.

KEYWORDS

fractions, intelligent tutoring system, mathematics, percentages, rational numbers

Practitioner notes

What is already known about this topic

- Longitudinal studies showed that basic mathematical skills predict fraction understanding.
- There is only limited evidence on whether similar predictions can be observed for percentage understanding—in general and within intelligent tutoring systems.
- Process data from such intelligent tutoring systems can be leveraged to pursue both educational research questions and optimizing digital learning software.
- Problems involving percentages typically are word problems requiring multiplications and/or divisions.

What this paper adds

- Similar to the case of fractions, students' performance on advanced arithmetic, measurement units and geometry significantly predicted performance with percentages.
- Students' performance with fractions also predicted performance with percentages significantly.
- A psychological network analysis was applied to evaluate specific interdependencies between a range of subtopics (eg, *Multiplying and dividing fractions, Adding and subtracting fractions* and *Calculating with percentages*).
- Fraction word problems and fraction problems involving multiplication/division turned out to be the best predictors of understanding percentages.

Implications for practice and/or policy

- When facing difficulties with percentages, revision of previous mathematical concepts sharing similar features (eg, fraction word problems, fraction/natural number multiplication/division problems) may be advised.
- Software developers may consider implementing such data-driven revision recommendations for students facing difficulties within intelligent tutor systems.
- Psychological network analysis can be utilized as a learning analytics method for easy-to-access visualizations illustrating relationships between a large range of different subtopics.

INTRODUCTION

Rational numbers frequently occur in our daily lives. For instance, we order 1/4 of a pizza for lunch; we pay 1/2 the price when things are on sale; or we get 30% off on summer sales. This is substantiated by a recent survey including over 2000 US citizens indicating that over

68% of them reported to frequently operate with rational numbers (Handel, 2016). However, despite the relevance of understanding rational numbers in our everyday lives, children, adolescents, college students and even educated adults face considerable difficulties when operating with them (Gigerenzer & Gaissmaier, 2011; Kallai & Tzelgov, 2012; Liu et al., 2014; Lortie-Forgues et al., 2015; Obersteiner et al., 2013, 2015; Siegler & Lortie-Forgues, 2015, 2017; Stigler et al., 2010).

Interestingly, over the last decade, a growing body of research has focused on the understanding of fractions—one form of rational numbers (Bailey et al., 2014; Booth et al., 2014; Booth & Newton, 2012; Braithwaite et al., 2017, 2018, 2022; Braithwaite & Siegler, 2020, 2023; Bustamante et al., 2022; Hansen et al., 2017; Jordan et al., 2013; Liu & Braithwaite, 2022; Schneider & Siegler, 2010; Siegler et al., 2011, 2013; Tian et al., 2020; Tian & Siegler, 2017). However, surprisingly few studies thus far examined the understanding of percentages as another form of rational numbers (but see Erdem et al., 2021; Gay & Aichele, 1997; Guiler, 1946a, 1946b; Jitendra & Star, 2012; Lembke & Reys, 1994; Parker & Leinhardt, 1995; Pöhler et al., 2021; Pöhler & Prediger, 2015; Prediger & Neugebauer, 2023; Siegler & Tian, 2022).

This seems surprising as recent research repeatedly indicated fraction understanding to robustly predict not only later mathematical achievement (Bailey et al., 2012; Barbieri et al., 2021; Booth & Newton, 2012; Siegler et al., 2012) but also academic and life prospects more generally (Duncan et al., 2007; Murnane et al., 1995; National Mathematics Advisory Panel, 2008; Ritchie & Bates, 2015; Rivera-Batiz, 1992). Thus, a better comprehension of the predictors of rational number understanding seems critical for mathematical development in general. Interestingly, previous research also indicated that fraction understanding is predicted significantly by more basic mathematical skills including numbers and arithmetic operations, suggesting that students with better basic mathematical skills experience fewer difficulties understanding fractions (Bailey et al., 2014; Hansen et al., 2015, 2017; Hecht & Vagi, 2010; Jordan et al., 2013; Spitzer & Moeller, 2022; Vukovic et al., 2014). However, it remains unclear whether percentage understanding can also be predicted by basic mathematical skills found to predict fraction understanding. Moreover, it remains unclear whether understanding one form of rational numbers (fractions) predicts the understanding of another form of rational numbers (percentages).

To evaluate this, we considered a large-scale dataset (2.798 Dutch students; ≈4.1 million problems) from an intelligent tutoring system (henceforth ITS). Within this ITS, students worked through a set of different basic mathematical topics, followed by fractions and then percentages. In a two-step procedure, we first investigated whether percentage understanding was predicted by basic mathematical skills as well as fraction understanding, to gain knowledge on which mathematical topics (ie, *Geometry*, *Basic Arithmetic*, *Measurement Units*, *Advanced Arithmetic or Fractions*) best predicted percentage understanding. In a second step, we then looked at our dataset more closely and employed a psychological network analysis (Epskamp et al., 2018; Epskamp & Fried, 2018) to identify whether there were *specific* subtopics on the predictor side (eg, fraction multiplication vs. fraction addition) that predicted percentage understanding.[1]

As such, our analyses not only allow us to draw conclusions about the predictive power of basic mathematical skills and fraction understanding when it comes to percentage understanding. Instead, they also provide valuable insights into how to foster learning about percentages within ITSs—by evaluating interdependencies between specific subtopics and, based on this evaluation, recommending targeted revisions of those subtopics that demonstrate the highest predictive value for addressing difficulties in percentage understanding. Thereby, this study will contribute valuable insights into the understanding of rational numbers over and beyond fractions by offering actionable recommendations for both the broader educational context as well as the further development of ITSs.

In the following, we first review previous research on understanding rational numbers discussing specifics for understanding percentages, and recent research on fractions. We then provide a brief overview of ITSs including examples of previous research leveraging data obtained from ITS to answer a research question on the development of mathematical skills as well as our understanding of how to improve such systems. Finally, we outline the details of the present study.

Previous research on rational numbers

Tian and Siegler (2018) reviewed existing research on rational numbers and reported that the vast majority of studies on rational numbers considered fractions (see below), with only a few studies on decimals, and hardly any research on percentages (Erdem et al., 2021; Gay & Aichele, 1997; Guiler, 1946a, 1946b; Jitendra & Star, 2012; Lembke & Reys, 1994; Parker & Leinhardt, 1995; Pöhler et al., 2021; Pöhler & Prediger, 2015; Prediger & Neugebauer, 2023; Siegler & Tian, 2022). These studies typically investigated students' difficulties when understanding percentages and strategies on how students solve tasks including percentages.

Because of the imbalance of research on the different forms of rational numbers, Tian and Siegler (2018) suggested that more research needs to be conducted on the development of percentage understanding, including research considering transfer effects between different forms of rational numbers. Regarding the latter, one may argue that different forms of rational numbers should build on similar basic mathematical skills, as different forms of rational numbers refer to the same magnitude only expressed differently (eg 1/2 = 50%). Based on this reasoning, one may hypothesize that significant predictors for performance on one form of rational numbers (eg, fractions) may similarly predict performance on another form of rational numbers (such as percentages). Nevertheless, there may be specificities across problems involving different forms of rational numbers.

Previous research on percentages

Interestingly, problems involving percentages may not only differ from fractions regarding the representational form of rational numbers (eg, 1/2 vs. 50%) but also the type of problem. This is further illustrated by Siegler and Tian (2022) who reported on a recent analysis of problems in two major US textbooks suggesting that problems including percentages differed from fraction or decimal problems in several respects. First, percentages are typically introduced using problems that require the conversion of fractions or decimals to percentages (eg, Write as a percent: 3/10 = ?; 0.7 = ?). Second, problems on calculating with percentages typically comprise three variables (the base value, the percent and the amount) with two of these variables given and students having to calculate the third (eg, 'What is 80% of 250g? Calculate the amount!'; also see Pöhler & Prediger, 2015; Prediger & Neugebauer, 2023). As such, these percentage problems typically come with a focus on multiplication and division operations (here: 250/100*80 = 200) but less so with addition and subtraction operations (Siegler & Tian, 2022). Finally, most of the percentage problems are word problems (as the example illustrates; also see Jitendra & Star, 2012), whereas a substantial number of fraction and decimal problems also comprise mere calculation problems involving adding or subtracting fractions (eg, calculate: 1/2 + 1/3 = ?).

Thus, one may argue that the similarity in features between mathematical problems of different topics (eg, word problems or a focus on multiplication and division) should also influence predictors for understanding percentages. However, to the best of our knowledge, there are no longitudinal studies evaluating predictors of understanding percentages.

Therefore, we will briefly consider related evidence on the understanding of another form of rational numbers—fractions.

Previous research on fractions

To evaluate whether previous results on the prediction of fraction understanding are similar for understanding percentages, a first step may be to test whether fraction understanding and percentage understanding are predicted by the same basic mathematical skills (eg, for fractions see Bailey et al., 2014; Hansen et al., 2015, 2017; Hecht & Vagi, 2010; Jordan et al., 2013; Spitzer & Moeller, 2022; Vukovic et al., 2014; Wortha et al., 2023). For instance, there is compelling correlational and longitudinal evidence indicating that students with better basic number and arithmetic skills do better in learning fractions (Bailey et al., 2014; Hansen et al., 2015, 2017; Hecht & Vagi, 2010; Jordan et al., 2013; Vukovic et al., 2014). For example, Jordan et al. (2013) observed that third graders' natural number knowledge and arithmetic skills significantly predicted their fraction understanding by the end of fourth grade. Similarly, Hansen et al. (2015) reported that fifth graders' basic arithmetic skills predicted their sixth-grade fraction understanding. Importantly, these studies relied on data from in-person testing. However, a substantial number of students currently engage with ITSs for learning mathematics (Meeter, 2021; Spitzer et al., 2023; Tomasik et al., 2020; Van Schoors et al., 2021). Accordingly, it is important to consider learning mathematics within ITSs in more detail.

Intelligent tutoring systems for learning mathematics

ITSs for learning mathematics are educational tools developed to ease learning through personalized tutoring by leveraging a broad collection of adaptive features, such as providing students with tailored content, feedback and help features (eg, Anderson et al., 1985; Koedinger et al., 1997, 2023; Mavrikis & Holmes, 2019; Ritter et al., 2007; VanLehn, 2011). In so doing, ITSs seek to facilitate learning by providing sufficient scaffolding as well as immediate feedback for students (eg, Rittle-Johnson & Koedinger, 2005) and incorporating interactive tools or gamification elements (eg gaining stars for correct answers) to encourage and maintain active participation and engagement (Mavrikis et al., 2019; Mavrikis & Holmes, 2019; Ninaus et al., 2023; Plass et al., 2015; Spitzer et al., 2023).

Importantly, these systems record continuous data as students solve problems, generating extensive datasets that can be utilized to enhance our comprehension of developmental processes (Grawemeyer et al., 2017; Mavrikis et al., 2022; Rau et al., 2009, 2016; Rittle-Johnson & Koedinger, 2005; Rittle-Johnson & Koedinger, 2009) and optimize digital learning software to cater to students' needs (Rienties et al., 2017; Ritter et al., 2007). For instance, research on the effect of scaffolds in ITSs showed that incorporating both conceptual and procedural scaffolds enhanced students' fraction learning (Rittle-Johnson & Koedinger, 2005). In particular, Rittle-Johnson and Koedinger (2005) found that visual aids for conceptual skills and step-by-step guidance for procedural skills improved students' ability to add and subtract fractions. Similarly, Mavrikis et al. (2022) demonstrated that students using an ITS combined with an exploratory learning environment achieved better outcomes when learning fractions than those using an ITS focused solely on procedural skills, highlighting the effectiveness of integrated learning approaches for the acquisition of fraction understanding.

Additionally, using data from a large ITS for learning mathematics, Spitzer and Moeller (2022) observed that basic mathematical skills, such as arithmetic and comprehension of measurement units, significantly predicted fraction understanding—similar to the results of studies relying on in-person testing (see above). Importantly, however, these results were observed for learning fractions, and comparable research on understanding percentages is largely missing so far—which will be addressed in the current study.

The present study

As outlined above, percentages are just another form of representing rational numbers. However, problems involving percentages often differ from problems involving fractions. Thus, it is an open question whether basic mathematical skills observed to predict fraction understanding within an ITS also predict percentage understanding. In addition, it remains unknown, to the best of our knowledge, whether understanding one form of rational numbers (ie, fractions) facilitates understanding another form of rational numbers (ie, percentages).

Furthermore, the results by Spitzer and Moeller (2022) solely considered the prediction of average performance on a large range of fraction problems based on average performance on basic mathematical skills (ie, geometry, arithmetic and measurement units). However, one may ask whether specific subtopics of previous mathematical topics (eg, fraction/natural number word problems or fraction/natural number multiplication/division problems) predict performance on percentages differentially. Accordingly, this study sought to evaluate whether specific mathematical subtopics predict performance on calculating with percentages (the operationalization of percentage understanding in this study) better than others. If so, this would provide researchers with detailed insights regarding developmental trajectories of percentage understanding. In addition, these insights could also be relevant for software developers to implement mechanisms which may suggest specific subtopics to students to revise when facing difficulties with percentages.

Accordingly, we first evaluated whether students' performance in basic mathematical skills previously found to predict fraction performance, also predicted students' performance in percentages. We then investigated whether this prediction was also observed when considering fraction performance (as a first instance of dealing with rational numbers) as a predictor of percentage performance. Finally, we evaluated whether there are specific subtopics that predict performance in percentages best.

To pursue these research questions, we analysed longitudinal data from the ITS *bettermarks* collected in the Netherlands. In particular, we included data regarding students working on the following mathematical topics implemented in *bettermarks*: (i) *Geometry*; (ii) *Basic Arithmetic*; (iii) *Measurement Units*; (iv) *Advanced Arithmetic*; (v) *Fractions* and (vi) *Calculating with percentages*. Students' average performance on the first four topics served as indicators of students' basic mathematical skills. Students' average performance on the *Fractions* topic and the *Calculating with percentages* topic served as the operationalization for fraction understanding and percentage understanding respectively.

Our analyses comprised two major steps. In the first step, we carried out three analyses which were based on students' average performance in each of the considered topics. Thereby, we sought to replicate previous findings indicating that basic mathematical skills significantly predict performance in fractions with the present dataset (cf. Spitzer & Moeller, 2022). Additionally, we aimed to extend this work by evaluating whether these four basic mathematical skills also predicted students' performance in percentages. In a third analysis, we further expanded this to test whether these basic mathematical skills still predicted students' performance in percentages when controlling for the effect of students' performance in fractions.

In the second step, we operationalized the average subtopic performance of each considered topic by using the accuracy of problem sets from each subtopic for each student.

Based on this data, we computed a Pearson correlation matrix across all subtopics which served as the basis for the psychological network analysis (Epskamp et al., 2018; Epskamp & Fried, 2018). The psychological network then visualized this correlation matrix by depicting correlations between all problem sets (for a general description of this approach see Spitzer et al., 2024). Furthermore, we used the Fruchterman–Reingold algorithm for the layout of the psychological network. This algorithm is a force-directed graph layout algorithm used for visualizing networks (Fruchterman & Reingold, 1991). The main goal of this algorithm is to arrange nodes (vertices) of a network in a way that variables (visualized as nodes) that correlate relatively highly are positioned more closely together, whereas variables that correlate relatively low are spread out. Nodes are further connected with edges to further visualize correlations between variables. We introduced a threshold for correlations to only visualize correlations of $r=0.3$ or higher with edges. For better visualization, the size of nodes is scaled with the number of incoming edges, and the thickness of edges is scaled with the correlation strength. Finally, we also coloured subtopics of the same topics using the same colour to better identify clusters of topics with their subtopics. We applied the psychological network analysis to visualize which subtopics cluster together due to relatively higher correlations between problem sets.

Finally, we conducted another linear regression analysis that only considered subtopics with a correlation of $r=0.3$ or higher to evaluate which subtopics predicted the subtopic *Calculating with percentages* best. Some students considered in the first step did not complete these subtopics and thus the dataset for this final analysis was reduced to 1528 students.

Together, these analyses sought to address two research questions:

1. Can students' percentage understanding be predicted by their basic mathematical skills and fraction understanding?
2. Can specific subtopics be identified that predict performance in percentages best?

METHODS

General description of the ITS *bettermarks*

In 2008, *bettermarks*[2] was introduced as an ITS to facilitate learning mathematics through adaptive feedback and personalized instructions (a detailed description of the adaptive features implemented within the ITS is provided in the Supplementary Material; also see Spitzer et al., 2023; Spitzer & Moeller, 2024; Stapel et al., 2016; Whalen et al., 2024). While the ITS was first established for students (grades: 4–12; age range: 9–18) in Germany only, it now includes curriculum-based topics for several other countries, such as the Netherlands, Uruguay and South Africa.

In the Netherlands, a considerable number of classes have been instructed in mathematics with *bettermarks* (3088 classes; grade levels 7–11; age range: 12–17; 38,179 students who worked with *bettermarks* from January 2016 until March 2020). These classes usually worked through *bettermarks* systematically topic by topic[3] in the order in which these topics are implemented in *bettermarks* providing an ideal case for analysing learning trajectories of mathematical development. In the present study, we specifically considered data from students who used *bettermarks* in the Netherlands (see further inclusion criteria below).

Moreover, in the Dutch education system, students are placed in different educational tracks or pathways based on their abilities and interests. One of these tracks is called the *Hoger Algemeen Voortgezet Onderwijs* which translates to *Higher General Secondary Education*. The first year of this track is abbreviated with 1 HV within *bettermarks* and in the following. The age range for students in 1 HV is between 12 and 13 years. Here, we only considered problem sets of topics that were worked through by students within this track. We also only included problem sets students completed before the topic *Percentages* and six problem sets of the subtopic *Calculating with percentages* of the topic *Percentages* (see further inclusion criteria below). Note that we only considered this particular subtopic as we wanted to focus on a specific type of percentage problems in this analysis—namely *Calculating with percentages*.

All computed problem sets together with the error rate on these problem sets are logged. However, no sensitive personal information about students exists and all data is thus fully anonymized and cannot be traced back to individual students. Thus, data on mathematical performance (ie, error rate) can be analysed, however, without any associations to demographic information (eg, socioeconomic status, exact age or gender). *Bettermarks* shares the anonymized data for scientific purposes and the users of this study provided consent that their anonymized data are shared. Importantly, the study was conducted without any contribution of *bettermarks* to the study design or outcomes. Thus, this investigation was independent of *bettermarks* and the results may not represent the opinion of *bettermarks*.

Learning content within the ITS

The content of *bettermarks* used in the Netherlands is structured into more than 100 different topics. These topics cover different mathematical topics, such as natural number arithmetic, geometry, converting measurement units, fractions and percentages (see Figure 1). All these topics have a similar hierarchical structure. Each topic includes several specific subtopics (eg, the *Fractions* topic has the subtopic *Multiplying and dividing fractions*). Subtopics are composed of problem sets, each of which contains several mathematical problems. These problems may further comprise one or several solution steps (also see Figure 2). Importantly, all problems within the *bettermarks* system used in the Netherlands are based on the mathematics curriculum of the Netherlands. Table 1 lists all topics and subtopics included in this study. Note that we abbreviated topics as follows: *Geometry = G; Basic Arithmetic = B; Units = U; Advanced Arithmetic = A; Fractions = F; Calculating with percentages = P*.

The *bettermarks* system that is used in the Netherlands covers 7191 different problem sets. As we only considered topics of the 1 HV track and only considered problem sets that were calculated before the subtopic *Calculating with percentages* (see Figure 3 for problem example on this subtopic), the present analysis was based on 241 problem sets from six different topics that had 44 different subtopics (see Table 1 and inclusion criteria below).

Integration of the ITS in the class context

The ITS is typically used within the classroom context. That is, teachers assign either all problem sets of a topic or single problem sets (or a set of single problem sets) to their students within *bettermarks*. Furthermore, students may also study on their own and self-assign problem sets, and thus two different learning scenarios exist: working through mathematical problem sets assigned by teachers or self-selecting mathematical problem sets. However, 91% of all computed problem sets between January 2016 and March 2020 within *bettermarks* in the Netherlands were assigned by teachers, indicating that teacher-assigned problem sets comprise the most frequent use-case of *bettermarks*. Irrespective of how problem sets were assigned, students may work through these assignments in school or at home.

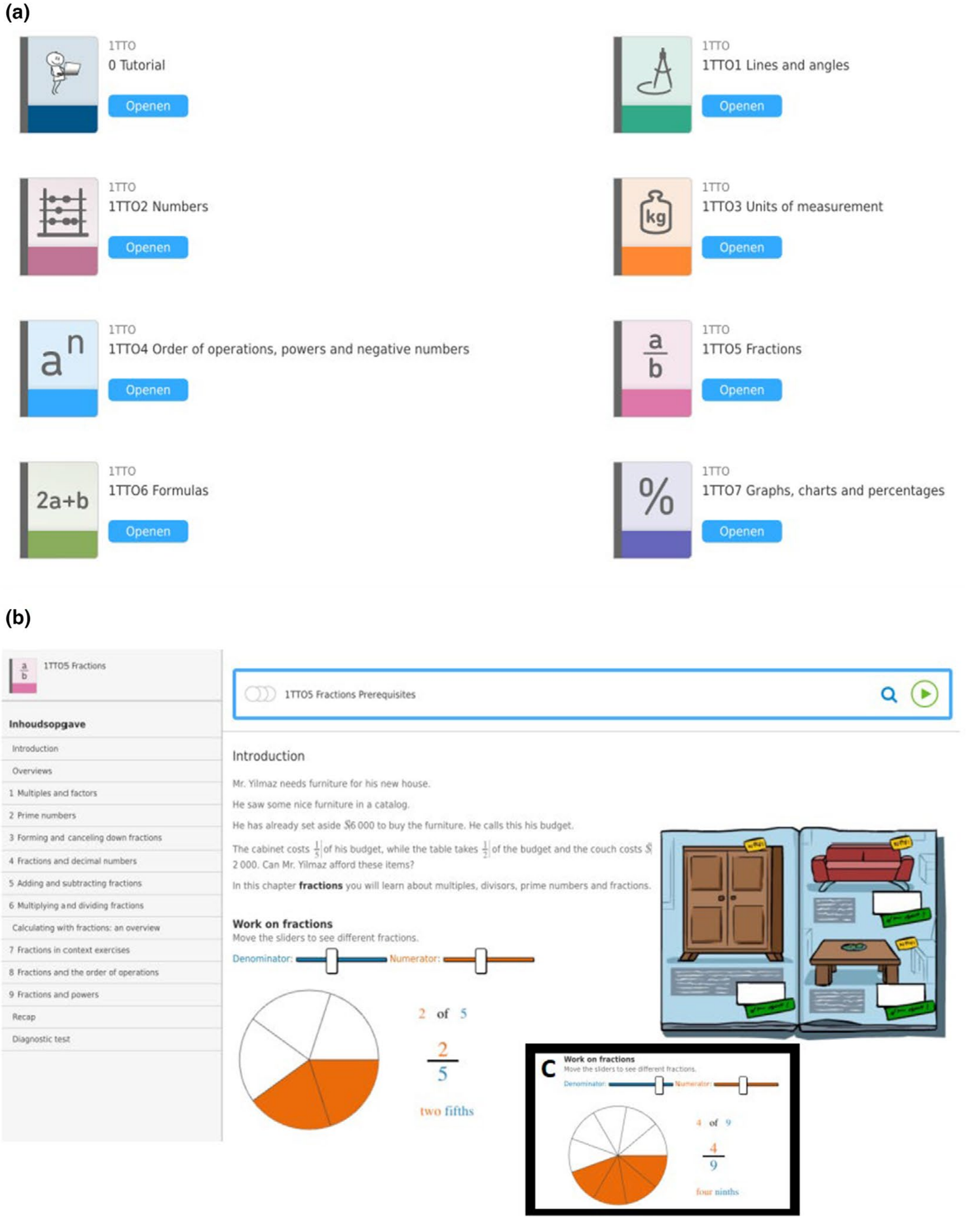

FIGURE 1 User interface of *bettermarks* and an example from the *Fractions* topic. (a) *bettermarks* user interface, (b) illustration of the topic *Fractions*, with subtopics on the left side and a short introduction as well as an interactive pie chart on the main window, and (c) an example of a worked-on interactive pie chart to explore fractions. Please note that both numerators (orange), as well as denominators (blue), can be explored by moving two different sliders to the left/right. The pie chart adjusts automatically by colouring less/more pie pieces in case the numerator is changed or by reducing/adding the number of pie pieces when the denominator is changed.

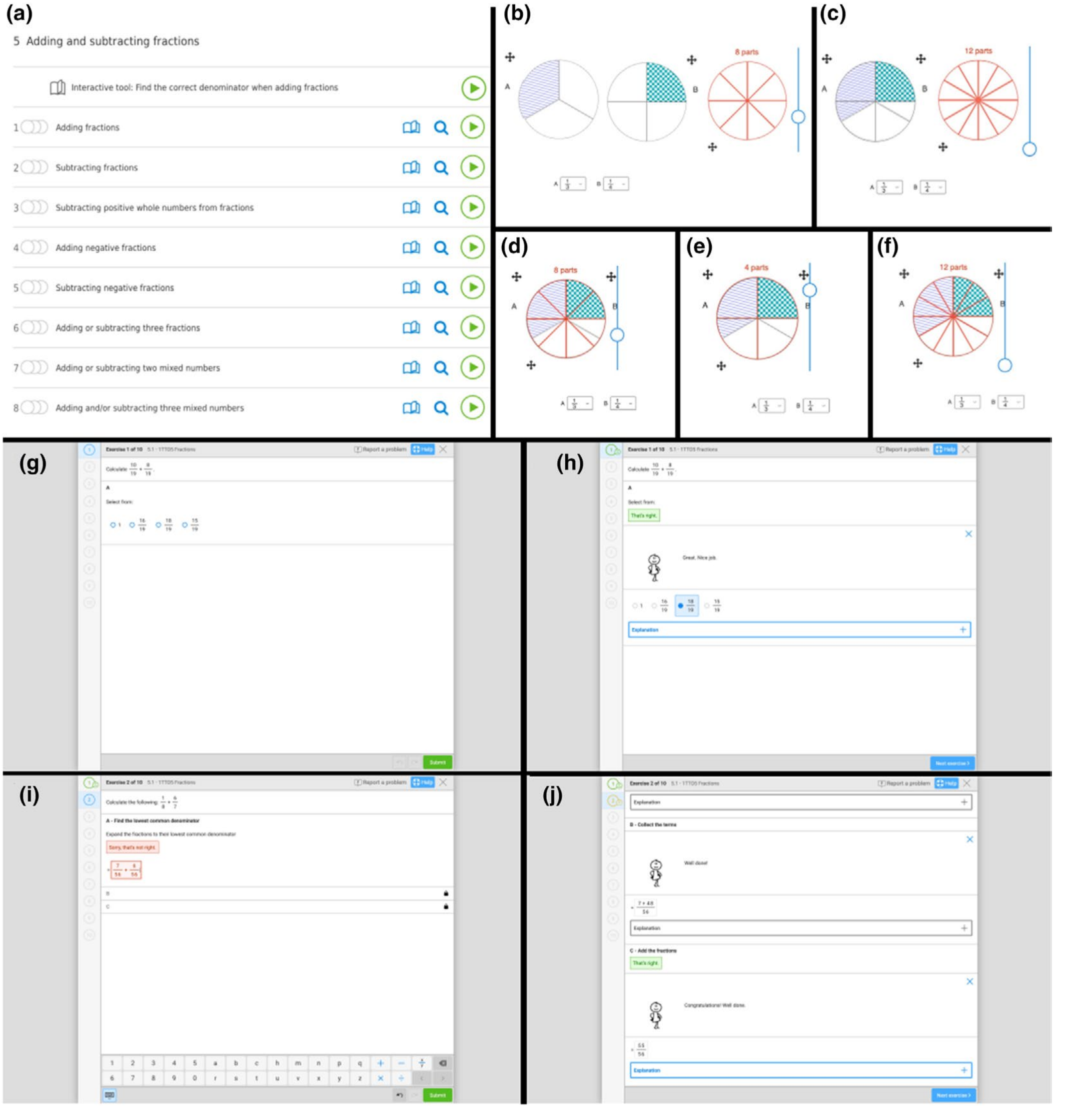

FIGURE 2 User interface of the subtopic *adding and subtracting fractions* (a) and examples of working with *bettermarks* (b–j). (a) The user interface with an exploratory tool to help students find common denominators as well as the problem sets of Subtopic 5 *Adding and Subtracting Fractions* of the topic *Fractions*. (b–f) An interactive tool for fractions as an exploratory learning activity to find the common denominator of two fractions without feedback. (g) The first of 10 problems of the problem set *Adding Fractions*. This first problem requires adding fractions with common denominators. (h) Performance-contingent feedback after working through the problem given in (g). (i) Example of content-specific feedback after the incorrect answer was provided in the first step of a three-step problem. Note that the correct answer is not provided to students following this incorrect answer in the first step. Students may be able to solve step one through the provision of content-specific feedback or by looking at a hint. (j) Example of solving all three steps of the problem correctly on the second attempt. Note the green (h) and yellow (j) smiley in the upper left corner of each problem set. Green indicates that the problem set was solved on the first attempt. Yellow indicates that two attempts were needed.

Inclusion criteria

Several inclusion criteria were applied before the statistical data analysis. First, we included students who studied with *bettermarks* in the Netherlands between 1 January 2016 and 1 March 2020.

TABLE 1 Abbreviated topic names, subtopic names, caption ID, accuracy and computed problem sets (*n*).

Topic	Caption	Subtopic	Accuracy	*n*
G	0	Prerequisites	0.85	1031
G	1	Lines, parallel and perpendicular	0.92	15,668
G	2	How to draw line segments with a given length on your device and on paper	0.81	5409
G	3	Drawing and measuring angles	0.89	17,927
G	4	Calculations on angles	0.86	18,723
G	5	Circles, perpendicular bisectors and angle bisectors	0.88	12,867
G	6	Symmetry	0.93	17,409
B	0	Prerequisites	0.85	1254
B	1	Digits and numbers	0.87	10,808
B	2	Compare numbers	0.89	11,044
B	3	Calculating with large numbers	0.86	10,649
B	4	Calculating with decimal numbers	0.90	14,555
B	5	Rounding	0.88	23,645
B	6	Estimating	0.90	11,838
B	7	Diagnostic test	0.87	1604
U	0	Prerequisites	0.87	1274
U	1	Measures of length	0.86	19,508
U	2	Charts and scale	0.82	11,830
U	3	Measures of mass	0.86	17,583
U	4	Time	0.83	14,969
U	5	All kinds of units of measurement	0.81	7091
U	6	Diagnostic test	0.81	1518
A	0	Prerequisites	0.89	1117
A	1	Arithmetic concepts	0.91	6903
A	2	Order of operations	0.86	11,007
A	3	Word problems	0.88	10,565
A	4	Square numbers and square roots	0.85	8646
A	5	Powers	0.87	12,762
A	6	Negative numbers	0.84	9989
A	7	Add and subtract negative numbers	0.83	15,343
A	8	Calculating with negative numbers	0.79	10,623
A	9	Powers with a minus sign	0.71	2993
A	10	Diagnostic test	0.85	1433
F	0	Prerequisites	0.93	1124
F	1	Multiples and factors	0.78	11,343

TABLE 1 (Continued)

Topic	Caption	Subtopic	Accuracy	*n*
F	2	Prime numbers	0.82	6037
F	3	Forming and cancelling down fractions	0.91	19,735
F	4	Fractions and decimal numbers	0.93	13,352
F	5	Adding and subtracting fractions	0.82	11,960
F	6	Multiplying and dividing fractions	0.87	11,493
F	7	Fractions in context exercises	0.74	3482
F	8	Fractions and the order of operations	0.76	4818
F	9	Fractions and powers	0.89	1198
P	4	Calculating with percentages	0.91	10,660

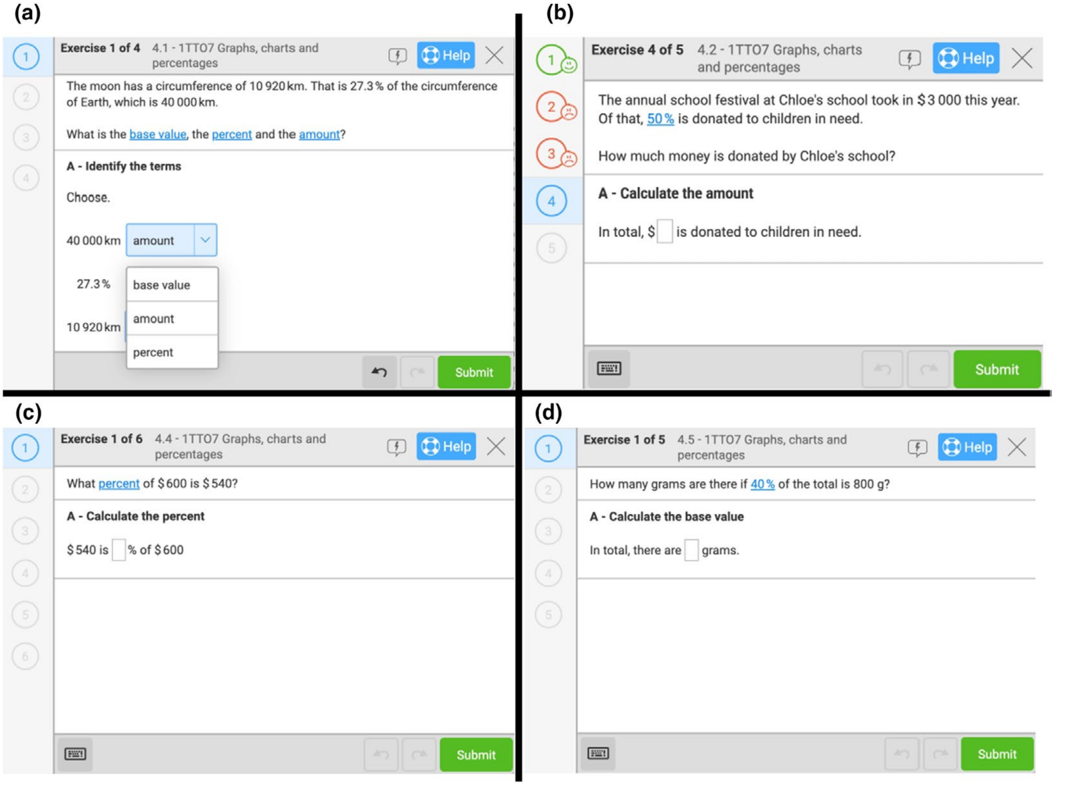

FIGURE 3 Problem examples of four different problem sets from the subtopic *Calculating with percentages* within *bettermarks*. (a) *Recognizing the percent, the amount and the base value*. (b) *Calculating the amount*. (c) *Calculating the percentage*. (d) *Calculating the base value*.

This period was selected since in mid-March 2020, schools were closed due to the outbreak of COVID-19 (Crompton et al., 2021; Engzell et al., 2021; Oliveira et al., 2021; Salas-Pilco et al., 2022; Spitzer et al., 2023; Spitzer & Moeller, 2024; St-Onge et al., 2022; Yan et al., 2021). Second, we only considered data from students who got problem sets assigned by their teachers, leaving out the rather small proportion of students who use the system to solve problems independently.

Third, we only considered students who worked through topics listed for the educational level 1 HV and only included topics of the 1 HV level which were worked through before the *Percentages* topic. These topics were *Geometry*, *Basic Arithmetic* including, for example, number line estimation, addition and subtraction problems, *Measurement Units*, and *Advanced Arithmetic*, including, calculating with negative numbers and order of operations (eg, $3+4*6=X$), and *Fractions*. We additionally included problem sets of the subtopic *Calculating* with *percentages*. Students completed all of their problem sets.

As teachers assigned problem sets categorized under 1 HV for students of age 12–13, there is good reason to assume that most of the students who worked on the considered problem sets were within the age range of 12–13 years. However, as no data on age are present, we can only speculate on the exact age range. A fourth inclusion criterion was that students had computed at least five problem sets in each of the considered topics or subtopics (ie, *Geometry, Basic Arithmetic, Measurement Units*, *Advanced Arithmetic, Fractions* and *Calculating with percentages*). Fifth, we only included problem sets that were computed 1000 times to obtain robust estimates for problem sets. Finally, we only considered students who worked through problems on *Geometry, Basic Arithmetic, Measurement Units* and *Advanced Arithmetic*, before they completed the *Fractions* topic (also see Figure S2). These students also had to work through *Fractions* before they worked through *Calculating with percentages*.

As students may repeat problem sets within *bettermarks* and thus may have produced more than one result on the same problem set, performance on each topic was calculated by considering students' best result on each of the computed problem sets within each topic. We then computed the average accuracy (ie, 1—error rate) for each student for each topic as an indicator of students' performance on them. It is important to note, however, that students' repetition rate was on average low (1.65 repetitions on average) and thus results were virtually identical when considering students' first attempts on problem sets instead of their best results (see Figure S1).

With these inclusion criteria applied, our final sample included 2836 students who worked through a total of 437,926 problem sets that stemmed from six different topics, 44 different subtopics and 241 different problem sets.

Data analysis

The data analysis was run in the R environment for statistical computing (R Core Team, 2013; RStudio Team, 2015). The psychological network analysis was conducted with the *igraph* package (Csardi & Nepusz, 2006).

In the first step, we operationalized the performance on a topic by computing the average accuracy on a topic for each student. The average accuracy of each student for each of the four topics on basic mathematical skills (ie, *Geometry, Basic Arithmetic, Measurement Units* and *Advanced Arithmetic*) served as predictor variables in all three models for each student (without any standardization procedure). Furthermore, as in Spitzer and Moeller (2022), we also considered the average day difference between each topic and the predictor topic to account for the number of days passed between topics as another covariate. For each of the following analyses, we ran one model that included this covariate and one model that did not include it. We compared the goodness of fit of each of these two models using the Bayesian information criterion (BIC). Smaller BICs indicate better model fits and a difference of 10 between BICs indicates a better model fit (Burnham & Anderson, 2004).

In Model 1a, performance on the *Fractions* topic served as the dependent variable, whereas performance on *Calculating with percentages* served as the dependent variable in Model 1b and Model 1c. In Model 1c, students' average performance on *Fractions* was added as another predictor variable.

A first analysis (Model 1a) was conducted to replicate previous observations that fraction understanding (outcome variable in Model 1) was predicted by prior performance on the following four topics: *Geometry, Basic Arithmetic, Measurement Units* and *Advanced Arithmetic* (cf. Spitzer & Moeller, 2022). A second analysis then predicted students' performance on *Calculating with percentages* (outcome variable in Model 1b) as a function of the same four predictors. A final third analysis (Model 1c) assessed the predictive power of fractions for *Calculating with percentages* to evaluate whether understanding of one form of rational numbers (fractions) also robustly predicts calculating with of other form of rational number (percentages). This analysis also controlled for previous mathematical performance (ie, *Geometry, Basic Arithmetic, Measurement Units* and *Advanced Arithmetic*).

All students were included in the three models conducted in the first step. However, as Model 1a did not include problem sets from the *Calculating with percentages* subtopic and Model 1b did not include problem sets from the *Fractions* topic, these two analyses comprised fewer problem sets, whereas Model 1c considered all calculated problem sets (Model 1a: 427,186 problem sets, Model 1b: 352,745 problem sets; Model 1c: 437,926 problem sets).

In the second step, we operationalized the average subtopic performance of each considered topic by using the accuracy of problem sets from each subtopic. We computed a Pearson correlation matrix which served as the basis for the psychological network analysis. The psychological network analysis then visualized this correlation matrix by depicting the correlations between all problem sets. Furthermore, we used the Fruchterman–Reingold algorithm for the layout of the psychological network analysis. This algorithm is a force-directed graph layout algorithm used for visualizing graphs or networks (Fruchterman & Reingold, 1991). The main goal of this algorithm is to arrange nodes (vertices) of a graph in such a way that variables (visualized as nodes) that correlate relatively highly are positioned close together, while variables that correlate relatively low are spread out. Nodes are further connected with edges to further visualize correlations between variables. However, we introduced a threshold for correlations to only visualize correlations of $r = 0.3$ or higher with edges. For better visualization, the size of nodes is scaled with the number of incoming edges, and the thickness of edges is scaled with the correlation strength. Finally, we also coloured subtopics of the same topics with distinct colours to better identify subtopics of the same topic. We applied the psychological network to visualize which subtopics cluster together due to relatively higher correlations between problem sets.

Finally, we conducted another linear regression model that only included subtopics with a correlation of 0.3 or higher with the subtopic to examine which subtopics predicted the subtopic *Calculating with percentages* best. Some students of the data analysed in the first step did not complete these subtopics and thus the dataset for this final analysis was reduced to 1528 students.

RESULTS

All three model comparisons of the first step revealed that the simpler models (without day differences as a covariate) had lower BICs suggesting better model fits (Model 1a: −7044 for the simple model vs. −7033 for the with day difference as a covariate; Model 1b: −4919 vs. −4906 favouring the simple model; Model 1c: −4965 vs. −4945 favouring the simple model).

The results of all three models (Model 1a–c) computed within our first step of analyses are depicted in Table 2. The results of each individual model are reported below. We report the unstandardized regression estimates indicated with *beta*, as well as the 95% confidence interval (95% CI), *t-values* and *p-values* in the text below. The psychological network is depicted in Figure 4. The results of the final linear regression analysis are reported in Table 3.

Model 1a: Fraction understanding as predicted by basic mathematical skills

We fitted a linear model to predict performance in *Fractions* with performance in *Geometry*, *Basic Arithmetic, Measurement Units* and *Advanced Arithmetic* (formula: *Fractions* ~ *Geometry* + *Basic Arithmetic* + *Measurement Units* + *Advanced Arithmetic*). The model explained a significant and substantial proportion of variance ($R^2 = 0.50$, $F(4, 2853) = 701.03$, $p < 0.001$, adj. $R^2 = 0.49$). The models' intercept, corresponding to *Geometry* = 0, *Basic Arithmetic* = 0, *Measurement Units* = 0 and *Advanced Arithmetic* = 0, was at 0.09 (95% CI [0.05, 0.12], $t(2853) = 4.64$, $p < 0.001$).

Within this model, the effect of the three topics *Geometry*, *Measurement Units* and *Advanced Arithmetic* was significant and positive indicating that better performance on these topics predicted better fraction understanding (*Geometry*: beta = 0.12, 95% CI [0.07, 0.16], $t(2853) = 4.84$, $p < 0.001$; *Measurement Units*: beta = 0.14, 95% CI [0.10, 0.19], $t(2853) = 6.13$, $p < 0.001$; *Advanced Arithmetic*: beta = 0.62, 95% CI [0.58, 0.66], $t(2853) = 30.24$, $p < 0.001$). However, *Basic Arithmetic* did not significantly predict *Fractions* (beta = 0.01, 95% CI [−0.04, 0.07], $t(2853) = 0.50$, $p = 0.615$). Comparable to Spitzer and Moeller (2022), this seemed to be due to a suppressor effect.

A further post hoc stepwise backward regression procedure revealed that when only including the first two topics (*Geometry* and *Basic Arithmetic*), *Basic Arithmetic* showed a significant effect on *Fraction* performance (beta = 0.37, 95% CI [0.32, 0.41], $t(2855) = 15.53$, $p < 0.001$) and suggests that influences of *Basic Arithmetic* on *Fractions* seemed to be subsumed by other topics such as *Measurement Units* and *Advanced Arithmetic*.

Model 1b: Calculating with percentages predicted by basic mathematical skills

We fitted a second linear model to predict performance in *Calculating with percentages* with performance in *Geometry*, *Basic Arithmetic, Measurement Units* and *Advanced Arithmetic* (formula: *Calculating with percentages* ~ *Geometry* + *Basic Arithmetic* + *Measurement Units* + *Advanced Arithmetic*). The model also explained a significant but moderate share of variance ($R^2 = 0.18$, $F(4, 2853) = 157.19$, $p < 0.001$, adj. $R^2 = 0.18$). The models' intercept, corresponding to *Geometry* = 0, *Basic Arithmetic* = 0, *Measurement Units* = 0 and *Advanced Arithmetic* = 0, was at 0.28 (95% CI [0.22, 0.34], $t(2853) = 9.49$, $p < 0.001$). Similar to the previous model that predicted performance in *Fractions*, performance on *Calculating with percentages* was significantly predicted by *Geometry, Measurement Units* and *Advanced Arithmetic* (*Geometry*: beta = 0.17, 95% CI [0.09, 0.24], $t(2853) = 4.41$, $p < 0.001$; *Measurement Units*: beta = 0.18, 95% CI [0.10, 0.25], $t(2853) = 4.78$, $p < 0.001$; *Advanced Arithmetic*: beta = 0.38, 95% CI [0.32, 0.45], $t(2853) = 11.83$, $p < 0.001$). Comparable to Model 1, *Basic Arithmetic* did not significantly predict *Calculating with percentages* (beta = −0.03, 95% CI [−0.12, 0.05], $t(2853) = -0.76$, $p = 0.447$). However, as in the previous analysis, a further post-hoc

TABLE 2 Regression estimates (beta), standard error of the mean (*SE*), *t*-value and *p*-value are reported for each model.

	Model 1a (fractions)				Model 1b (percent)				Model 1c (percent)			
Coeffcient	**Beta**	***SE***	***t*-Value**	***p*-Value**	**Beta**	***SE***	***t*-Value**	***p*-Value**	**Beta**	***SE***	***t*-Value**	***p*-Value**
Intercept	0.09	0.02	4.64	**<0.001**	0.28	0.03	9.49	**<0.001**	0.25	0.03	8.64	**<0.001**
Geometry	0.12	0.02	4.84	**<0.001**	0.17	0.04	4.41	**<0.001**	0.13	0.04	3.39	**0.001**
Basic arithmetic	0.01	0.03	0.50	0.615	−0.03	0.04	−0.76	0.447	−0.04	0.04	−0.90	0.370
Measurement units	0.14	0.02	6.13	**<0.001**	0.18	0.04	4.78	**<0.001**	0.13	0.04	3.46	**0.001**
Advanced arithmetic	0.62	0.02	30.24	**<0.001**	0.38	0.03	11.83	**<0.001**	0.16	0.04	4.49	**<0.001**
Fractions									0.36	0.03	12.34	**<0.001**
R^2/R^2 adjusted	0.496/0.495				0.181/0.179				0.222/0.221			

Note: *p* values that were significant were marked indicated in bold.

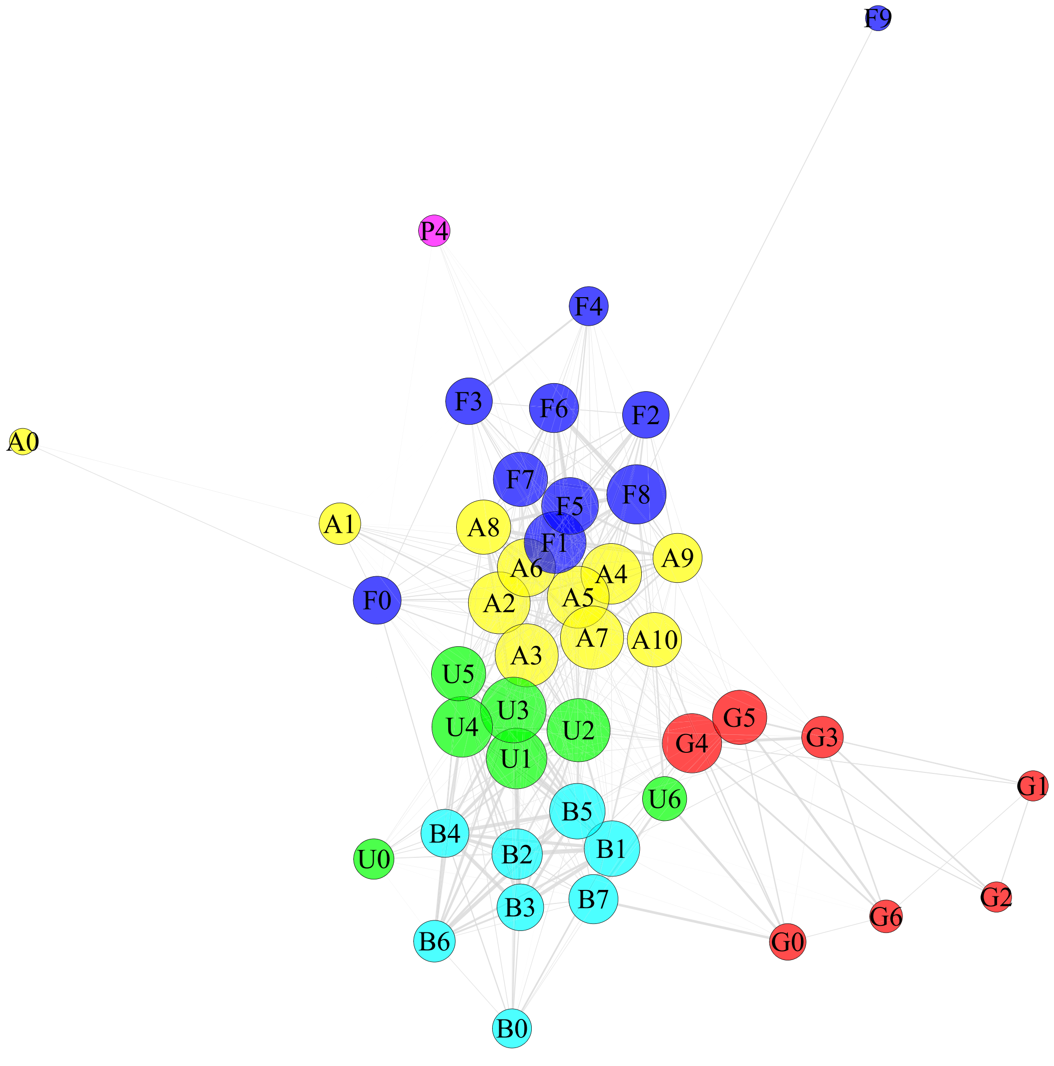

FIGURE 4 Psychological network depicting the interdependencies between all considered subtopics. Topics and subtopics are labelled with the abbreviated topic label and the respective subtopic number (eg, F6 refers to the 6th subtopic of *Fractions*; also see Table 1). Interestingly, it becomes evident that subtopics of the same topics clustered together indicating high intercorrelations within each topic. In addition, subtopics of the *Fractions* topic (blue nodes) were closest to the subtopic of *Calculating* with *percentages* (pink node), followed by *Advanced Arithmetic* (yellow nodes), and *Units* (green nodes) as well as *Geometry* (red nodes). Subtopics of *Basic Arithmetic* (turquoise nodes) were farthest away. Nodes scale with the number of incoming edges. Edges scale with the correlation degree with higher positive correlations increasing the edge width. Only correlations above 0.3 are depicted with edges.

regression analysis which only considered *Geometry* and *Basic Arithmetic* as predictors revealed a significant effect of *Basic Arithmetic* on *Calculating with percentages* ($b = 0.256$; $t = 7.92$; $p < 0.001$) indicating a suppressor effect. This result again suggests that the effect of *Basic Arithmetic* on *Calculating with percentages* was subsumed by other topics such as *Measurement Units* and *Advanced Arithmetic*. In general, these results indicated that results from predicting one form of rational numbers (ie, fractions) seem to generalize to predicting another form of rational numbers.

TABLE 3 Regression estimates (beta), standard error of the mean (*SE*), *t*-value and *p*-value are reported.

	Model 2			
Coeffcient	***B***	***SE***	***t*-Value**	***p*-Value**
Intercept	0.02	0.00	3.58	**<0.001**
F1: Multiples and factors	0.14	0.02	6.41	**<0.001**
F5: Adding and subtracting fractions	0.01	0.02	0.34	0.732
F6: Multiplying and dividing fractions	0.08	0.02	3.57	**<0.001**
F7: Fractions in context exercises	0.09	0.02	5.48	**<0.001**
F8: Fractions and the order of operations	0.04	0.01	2.75	**0.006**
R^2/R^2 adjusted	0.214/0.211			

Note: *p* values that were significant were marked indicated in bold.

Model 1c: *Calculating with percentages* predicted by basic mathematical skills and fraction understanding

We fitted a third linear model to predict performance in *Calculating with percentages* based on performance in *Geometry*, *Basic Arithmetic, Measurement Units, Advanced Arithmetic* and *Fractions* (formula: *Calculating with percentages ~ Geometry + Basic Arithmetic + Measurement Units + Advanced Arithmetic + Fractions*). The model explained a significant and moderate proportion of variance ($R^2 = 0.22$, $F(5, 2852) = 162.88$, $p < 0.001$, adj. $R^2 = 0.22$). In particular, *Geometry, Measurement Units*, *Advanced Arithmetic* and *Fractions* significantly predicted performance on *Calculating with percentages* (*Geometry*: beta = 0.13, 95% CI [0.05, 0.20], $t(2852) = 3.39$, $p < 0.001$; *Measurement Units*: beta = 0.13, 95% CI [0.05, 0.20], $t(2852) = 3.46$, $p < 0.001$; *Advanced Arithmetic*: beta = 0.16, 95% CI [0.09, 0.23], $t(2852) = 4.49$, $p < 0.001$; *Fractions*: beta = 0.36, 95% CI [0.30, 0.41], $t(2852) = 12.34$, $p < 0.001$). These results suggest that in addition to influences of basic mathematical skills, performance on one form of rational numbers (here fractions) transferred to another form of rational numbers (here percentages).

Again, *Basic Arithmetic* did not significantly predict *Calculating with percentages* (beta = −0.04, 95% CI [−0.12, 0.04], $t(2852) = -0.90$, $p = 0.370$). As the reduction of this model to *Geometry* and *Basic Arithmetic* only would be the same post hoc model as reported in the previous analyses (Model 1b), we did not run this model again.

Regression results summary

First, the regression results replicated previous findings of basic mathematical skills to predict performance in *Fractions* (Spitzer & Moeller, 2022). The results of our second and third model further suggested that performance in *Calculating with percentages* was precited by performance in *Geometry*, *Basic Arithmetic, Measurement Units, Advanced Arithmetic* and *Fractions*, with the strongest predictive power for *Fractions*, followed by *Advanced Arithmetic*.

However, these results stemmed from average performance scores of each topic. In addition, each of the predictor topics is further subdivided into subtopics within *bettermarks* (see Table 1), such as *Multiplying and dividing fractions* or *Fractions in context exercises*. Thus, variability between the predictive power of subtopics within each topic may exist and it remains unknown which specific subtopics best predict performance in *Calculating with percentages*. Thus, we sought to dig deeper and first depicted the correlations between all considered subtopics with a psychological network analysis.

Psychological network results

The psychological network is depicted in Figure 4. It shows that most subtopics constituting a topic (indicated by the same colour) were clustered together, suggesting high intercorrelations between subtopics of the same topic. In addition, subtopics were ordered in distance similar to the regression result of Model 1c, with most of the subtopics of the *Fractions* topic being related closest to the subtopic on *Calculating with percentages*, followed by subtopics on *Advanced Arithmetic, Measurement Units* and *Geometry*. Subtopics on *Basic Arithmetic* were farthest away from problem sets on *Calculating with percentages*. These results suggest that most of the subtopics followed the same correlational pattern as suggested by the linear regressions on average performance reported in the first step of analyses (Model 1a–c).

Finally, the edges of the psychological network indicated a correlation of $r=0.3$ or higher with *Calculating with percentages* (P4) for the following five subtopics of the *Fractions* topic: *Multiples and factors* (F1), *Adding and subtracting fractions* (F5), *Multiplying and dividing fractions* (F6), *Fractions in context exercises* (F7) and *Fractions and the order of operations* (F8). We tested the predictive power of the performance on these five subtopics for *Calculating with percentages* in a final linear regression analysis.

Which subtopics predict *Calculating with percentages* best?

Results of this linear regression analysis are listed in Table 3 and in the text below. We fitted a linear model to predict *Calculating with percentages* by *Multiples and factors* (F1), *Adding and subtracting fractions* (F5), *Multiplying and dividing fractions* (F6), *Fractions in context exercises* (F7) and *Fractions and the order of operations* (F8; formula: $P4 \sim F1+F5+F6+F7+F8$).

The model explained a significant and moderate proportion of variance ($R^2=0.21$, $F(5, 1522)=82.65$, $p<0.001$, adj. $R^2=0.21$). The model's intercept, corresponding to $F1=0$, $F5=0$, $F6=0$, $F7=0$ and $F8=0$, was at 0.02 (95% CI [7.18e−03, 0.02], $t(1522)=3.58$, $p<0.001$). Within this model, the effect of *Multiples and factors* (F1) was statistically significant and positive (beta $=0.14$, 95% CI [0.09, 0.18], $t(1522)=6.41$, $p<0.001$). The effect of *Adding and subtracting fractions* (F5) was non-significant but positive (beta <0.001, 95% CI [−0.03, 0.04], $t(1522)=0.34$, $p=0.732$). The effect of *Multiplying and dividing fractions* (F6) was significant and positive (beta $=0.08$, 95% CI [0.04, 0.12], $t(1522)=3.57$, $p<0.001$). The effect of *Fractions in context exercises* (F7) was statistically significant and positive (beta $=0.09$, 95% CI [0.06, 0.13], $t(1522)=5.48$, $p<0.001$). Finally, the effect of *Fractions and the order of operations* (F8) was statistically significant and positive (beta $=0.04$, 95% CI [0.01, 0.06], $t(1522)=2.75$, $p=0.006$).

A closer inspection of the predictor subtopics revealed that the subtopic *Multiples and factors* (F1) comprised mathematical problems that involved multiplication and division operations on natural numbers (eg, ‘Give the first 5 multiples of 11’.). This also indicated that the first subtopic of the *Fractions* topic did not comprise fractions but was rather an introduction to fractions by revisiting natural number multiplication and divisions. Furthermore, the two subtopics *Multiplying and dividing fractions* (F6) and *Fractions and the order of operations* (F8) involved multiplication and division operations on fractions. Moreover, the subtopic *Fractions in context exercises* (F7) comprised word problems on fractions.

Together, these results suggest that mathematical problems that involved multiplication and division operations on natural numbers (*Multiples and factors*) and fractions (*Multiplying and dividing fractions*, *Fractions and the order of operations*), as well as word problems involving fractions, significantly predicted students' performance on *Calculating with percentages*, with better performance on these subtopics leading to better performance on *Calculating with percentages*. Interestingly, students' performance on fraction problems involving addition and subtraction did not significantly predict their performance on *Calculating with percentages*. We further elaborate on this finding in the discussion below.

DISCUSSION

In this study, we investigated whether basic mathematical skills (ie geometry, basic arithmetic, measurement units and advanced arithmetic) predicted fraction understanding and percentage understanding. We also investigated whether fraction understanding predicted percentage understanding controlling for the influence of basic mathematical skills. Finally, we sought to identify specific subtopics that best predict percentage understanding. Therefore, we considered a large dataset (2836 students; 437,926 problem sets) from an ITS for learning mathematics. We present our main findings in the following sections before we discuss the implications of our results for researchers, practitioners, and software developers.

First, our results indicated that basic mathematical skills (ie, geometry, advanced arithmetic and measurement units) significantly and similarly predicted fraction understanding *and* percentage understanding. This finding corroborates the assumption that rational numbers (fractions and percentages) build similarly on basic mathematical skills attained before learning rational numbers: Students with a strong foundation in basic mathematical skills also performed better on rational numbers (at least fractions and percentages). These results provide, to the best of our knowledge, first insights into the longitudinal development of percentage understanding and as such extend previous studies that observed similar results for another form of rational numbers—fractions (Bailey et al., 2014; Hansen et al., 2015, 2017; Hecht & Vagi, 2010; Jordan et al., 2013; Spitzer & Moeller, 2022; Vukovic et al., 2014).

In addition, our results contribute empirical evidence to an ongoing discussion on whether understanding one form of rational numbers (eg, fractions) predicts another form of rational numbers (eg, percentages; Tian & Siegler, 2018) suggesting that fraction understanding positively and significantly predicts percentage understanding.

These two findings provide compelling evidence for the idea of a hierarchical nature of numerical development, however, they were based on students' *average* performance scores on each considered topic. Thus, we sought to dig deeper to further explore whether specific subtopics predicted percentage understanding best (see Figure 4). An inspection of all considered subtopics indicated that five fraction subtopics (ie, *Multiples and factors, Adding and subtracting fractions, Multiplying and dividing fractions, Fractions in context exercises*, and *Fractions and the order of operations*) correlated highest with *Calculating with percentages* (note the five edges depicting correlations between P4 and F1, F5, F6, F7 and F8 in Figure 4). A subsequent linear regression revealed that performance in four out of these five subtopics significantly predicted performance in *Calculating with percentages*, with better performance in *Multiples and factors, Adding and subtracting fractions, Multiplying and dividing fractions, Fractions in context exercises*, and *Fractions and the order of operations* predicting better performance in *Calculating with percentages* (see Table 3).

Notably, the features of the respective problems encountered within these four subtopics showed considerable similarities to features of problems in the *Calculating with percentages* subtopic. For instance, the subtopic *Calculating with percentages* predominantly consisted of word problems (similar to what has been reported; cf. Siegler & Tian, 2022) and problems requiring multiplication/division (cf. Siegler & Tian, 2022). Importantly, the fraction problems that structurally resemble those problems in the *Calculating with percentages* subtopic were identified as key predictors of performance in the *Calculating with percentages* subtopic (see the Results section: Which subtopics predict Calculating with percentages best?).

As such, our results on subtopics provide compelling evidence for practitioners and software developers that students facing difficulties with understanding percentages may benefit from revising problems on (a) natural number multiplications/divisions, (b) fraction multiplications/divisions or (c) fraction-based word problems. Moreover, software developers may consider implementing feedback and recommender structures within ITSs that specifically suggest these subtopics to students who face difficulties with understanding percentages.

Despite the promising results on the hierarchical nature of numerical development as well as providing practical guidance for practitioners and software developers on how to ease learning about percentages, there are limitations to be considered when interpreting the results of our study. First, we only had access to the data provided by *bettermarks* which only comprises performance data on problem sets students worked through within *bettermarks* but no other demographical or personal data. In other words, we do not know what else students were exposed to, what other problem sets students worked through outside *bettermarks*, and how well students performed in school. Furthermore, we do not know when students were exposed to fractions and percentages for the first time and when teachers first introduced the concept of percentages to their students. Nevertheless, we assume that teachers assigned problems on percentage understanding to their students shortly after they introduced percentages as a topic. The results presented in Figure S2 indicate that students worked through the topics in the order as they are presented in *bettermarks*. The fact that we only considered data from students who got problem sets assigned by their teachers further supports the assumption that students and teachers used *bettermarks* systematically, working through problems on percentage understanding after working through the other topics.

Another limitation of our study regards the second step of analysis. Not all students worked through all problem sets of each topic[4] and subtopic.[5] Thus, averages on topics and subtopics may have stemmed from different problem sets. We only considered problem sets that were computed at least 1000 times to obtain robust estimates for topics and subtopics, but some problem sets and subtopics were still computed more often than others (see Table 1). Nevertheless, future studies with well-controlled designs are needed to substantiate our results.

CONCLUSION

In this study, we evaluated longitudinal predictors for understanding percentages. We observed that basic mathematical skills (ie, arithmetic, measurement units and geometry) as well as fraction understanding significantly predicted percentage understanding. Closer inspection of the structural features of percentage problems suggested that these problems primarily consisted of word problems involving multiplications or division operations. In a next step, we found that problems with these features but on other topics that were worked through before percentages (eg, fraction-based word problems or natural number multiplications and fractions multiplications/divisions) were the best predictors of percentage understanding. This suggests that students facing difficulties with percentages within ITSs may specifically benefit from revising problems from other more basic mathematical topics which share specific features (ie, word problems and requiring multiplication/division). As such, our results point software developers for ITSs towards designing—or at least recommending students to revise—problems that share similar features of percentage problems, to facilitate learning about percentages or help students who face difficulties with percentages. This way, our results seem a first step towards more in-depth research on how to ease learning percentages within ITSs as well as how to best design ITS based on data-driven evidence.

ACKNOWLEDGEMENTS

We thank *Bettermarks* for sharing their data with us. The opinions expressed in this publication are those of the authors and do not necessarily reflect the views of *Bettermarks*. *Bettermarks* had no role in study design, data analysis, decision to publish, or preparation of the manuscript. Open Access funding enabled and organized by Projekt DEAL.

FUNDING INFORMATION

This research did not receive funding.

CONFLICT OF INTEREST STATEMENT

None of the authors have a conflict of interest to disclose.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

ETHICS STATEMENT

In this study, we are reporting a retrospective study of archived data which does not contain any sensitive user data. Thus, it is not possible to track the data back to any software user.

ENDNOTES

[1] Psychological network analysis visualizes correlation patterns by depicting variables as nodes and correlations between variables as edges with higher correlations being reflected by nodes being placed closer to each other and edges being stronger. Applied to the context of learning mathematics, psychological network analysis allows to illustrate a possible framework for understanding how students' performance on different subtopics are interrelated.

[2] *Bettermarks* permitted us to be mentioned explicitly and to share snapshots of their user interface.

[3] Note that students may select problem sets from topics in any order and that teachers may also assign problem sets in any order. Thus, students do not have to work through the sequence in which topics are listed. Here, we only included students who worked through the topics in the order that appears within *bettermarks* (see analyses below) which were about 87% of all considered students (see inclusion criteria).

[4] Students worked through at least 5 problem sets of each topic.

[5] Students worked through at least 1 problem set of each subtopic.

REFERENCES

Anderson, J. R., Boyle, C. F., & Reiser, B. J. (1985). Intelligent tutoring systems. *Science*, *228*, 246–255. https://doi.org/10.4324/9781315617572

Bailey, D. H., Hoard, M. K., Nugent, L., & Geary, D. C. (2012). Competence with fractions predicts gains in mathematics achievement. *Journal of Experimental Child Psychology*, *113*(3), 447–455. https://doi.org/10.1016/j.jecp.2012.06.004

Bailey, D. H., Siegler, R. S., & Geary, D. C. (2014). Early predictors of middle school fraction knowledge. *Developmental Science*, *17*(5), 775–785. https://doi.org/10.1111/desc.12155

Barbieri, C. A., Young, L. K., Newton, K. J., & Booth, J. L. (2021). Predicting middle school profiles of algebra performance using fraction knowledge. *Child Development*, *92*(5), 1984–2005. https://doi.org/10.1111/cdev.13568

Booth, J. L., & Newton, K. J. (2012). Fractions: Could they really be the gatekeeper's doorman? *Contemporary Educational Psychology*, *37*(4), 247–253. https://doi.org/10.1016/j.cedpsych.2012.07.001

Booth, J. L., Newton, K. J., & Twiss-Garrity, L. K. (2014). The impact of fraction magnitude knowledge on algebra performance and learning. *Journal of Experimental Child Psychology*, *118*(1), 110–118. https://doi.org/10.1016/j.jecp.2013.09.001

Braithwaite, D. W., McMullen, J., & Hurst, A. M. (2022). Cross-notation knowledge of fractions and decimals. *Journal of Experimental Child Psychology*, *213*, 105–210.

Braithwaite, D. W., Pyke, A. A., & Siegler, R. S. (2017). A computational model of fraction arithmetic. *Psychological Review*, *2016*, 603–625.

Braithwaite, D. W., & Siegler, R. (2020, March). Putting fractions together. *Journal of Educational Psychology*, *113*, 556–571. https://doi.org/10.1037/edu0000477

Braithwaite, D. W., & Siegler, R. S. (2023, July). A unified model of arithmetic with whole numbers, fractions, and decimals. *Psychological Review*, *131*, 431–455. https://doi.org/10.1037/rev0000440

Burnham, K. P., & Anderson, D. R. (2004). Multimodel inference: understanding AIC and BIC in model selection. *Sociological methods & research*, *33*(2), 261–304.

Braithwaite, D. W., Tian, J., & Siegler, R. S. (2018). Do children understand fraction addition? *Developmental Science*, *21*(4), e12601.

Bustamante, A. S., Begolli, K. N., Alvarez-Vargas, D., Bailey, D. H., & Richland, L. E. (2022). Fraction ball: Playful and physically active fraction and decimal learning. *Journal of Educational Psychology*, *114*(6), 1307–1320. https://doi.org/10.1037/edu0000714

Crompton, H., Burke, D., Jordan, K., & Wilson, S. W. G. (2021). Learning with technology during emergencies: A systematic review of K-12 education. *British Journal of Educational Technology*, *52*(4), 1554–1575. https://doi.org/10.1111/bjet.13114

Csardi, G., & Nepusz, T. (2006). The igraph software package for complex network research. *Inter Journal of Complex Systems*, *1695*, 1–9.

Duncan, G. J., Claessens, A., Magnuson, K., Klebanov, P., Pagani, L. S., Feinstein, L., Engel, M., Brooks-gunn, J., Sexton, H., Duckworth, K., & Japel, C. (2007). School readiness and later achievement. *Developmental Psychology*, *43*(6), 1428–1446. https://doi.org/10.1037/0012-1649.43.6.1428

Engzell, P., Frey, A., & Verhagen, M. D. (2021). Learning loss due to school closures during the COVID-19 pandemic. *Proceedings of the National Academy of Sciences of the United States of America*, *118*(17), e2022376118. https://doi.org/10.1073/PNAS.2022376118

Epskamp, S., Borsboom, D., & Fried, E. I. (2018). Estimating psychological networks and their accuracy: A tutorial paper. *Behavior Research Methods*, *50*(1), 195–212. https://doi.org/10.3758/s13428-017-0862-1

Epskamp, S., & Fried, E. I. (2018). A tutorial on regularized partial correlation networks. *Psychological Methods*, *23*(4), 617–634. https://doi.org/10.1037/met0000167

Erdem, E., Gökkurt, B., & Soylu, Y. (2021). Seventh graders' strategies for shading the percentages of geometric shapes. *International Journal of Mathematical Education in Science and Technology*, *54*, 801–826. https://doi.org/10.1080/0020739X.2021.1976854

Fruchterman, T. M. J., & Reingold, E. M. (1991). Graph drawing by force-directed placement. *Software-Practice and Experience*, *21*(11), 1129–1164. https://doi.org/10.1007/978-3-319-64471-4_31

Gay, A. S., & Aichele, D. B. (1997). Middle school students´ understanding of number sense related to percent. *School Science and Mathematics*, *97*(1), 27–36.

Gigerenzer, G., & Gaissmaier, W. (2011). Heuristic decision making. *Annual Review of Psychology*, *62*, 451–482. https://doi.org/10.1146/annurev-psych-120709-145346

Grawemeyer, B., Mavrikis, M., Holmes, W., Gutiérrez-Santos, S., Wiedmann, M., & Rummel, N. (2017). Affective learning: Improving engagement and enhancing learning with affect-aware feedback. *User Modeling and User-Adapted Interaction*, *27*(1), 119–158. https://doi.org/10.1007/s11257-017-9188-z

Guiler, W. S. (1946a). Difficulties encountered in percentage by college freshmen. *The Journal of Educational Research*, *40*(2), 81–95.

Guiler, W. S. (1946b). Difficulties in percentage encountered by ninth-grade pupils. *The Elementary School Journal*, *46*(10), 563–573.

Handel, M. J. (2016). What do people do at work? A profile of U.S. jobs from the survey of workplace skills, technology, and management. *Journal of Labor Market Research*, *49*, 177–197. https://doi.org/10.1007/s12651-016-0213-1

Hansen, N., Jordan, N. C., Fernandez, E., Siegler, R. S., Fuchs, L., Gersten, R., & Micklos, D. (2015). General and math-specific predictors of sixth-graders' knowledge of fractions. *Cognitive Development*, *35*, 34–49. https://doi.org/10.1016/j.cogdev.2015.02.001

Hansen, N., Jordan, N. C., & Rodrigues, J. (2017). Identifying learning difficulties with fractions: A longitudinal study of student growth from third through sixth grade. *Contemporary Educational Psychology*, *50*, 45–59. https://doi.org/10.1016/j.cedpsych.2015.11.002

Hecht, S. A., & Vagi, K. J. (2010). Sources of group and individual differences in emerging fraction skills. *Journal of Educational Psychology*, *102*(4), 843–859. https://doi.org/10.1037/a0019824

Jitendra, A. K., & Star, J. R. (2012). An exploratory study contrasting high- and low-achieving students' percent word problem solving. *Learning and Individual Differences*, *22*(1), 151–158. https://doi.org/10.1016/j.lindif.2011.11.003

Jordan, N. C., Hansen, N., Fuchs, L. S., Siegler, R. S., Gersten, R., & Micklos, D. (2013). Developmental predictors of fraction concepts and procedures. *Journal of Experimental Child Psychology*, *116*(1), 45–58. https://doi.org/10.1016/j.jecp.2013.02.001

Kallai, A. Y., & Tzelgov, J. (2012). When meaningful components interrupt the processing of the whole: The case of fractions. *Acta Psychologica*, *139*(2), 358–369. https://doi.org/10.1016/j.actpsy.2011.11.009

Koedinger, K. R., Anderson, J. R., Hadley, W. H., & Mark, M. A. (1997). Intelligent tutoring goes to school in the big city. *International Journal of Artificial Intelligence in Education*, *1*(1), 30–43.

Koedinger, K. R., Carvalho, P. F., Liu, R., & Mclaughlin, E. A. (2023). An astonishing regularity in student learning rate. *Proceedings of the National Academy of Sciences*, *120*(13), e2221311120. https://doi.org/10.1073/pnas

Lembke, L. O., & Reys, B. J. (1994). The development of, and interaction between, intuitive and school-taught ideas about percent. *Journal for Research in Mathematics Education*, *25*(3), 237–259.

Liu, Q., & Braithwaite, D. W. (2022). Affordances of fractions and decimals for arithmetic. *Journal of Experimental Psychology: Learning Memory and Cognition*, *49*(9), 1459.

Liu, R., Zong, M., DIng, Y., & Zhang, D. (2014). Concept development of decimals in Chinese elementary students: A conceptual change approach. *School Science and Mathematics*, *114*(7), 326–338.

Lortie-Forgues, H., Tian, J., & Siegler, R. S. (2015). Why is learning fraction and decimal arithmetic so difficult? *Developmental Review*, *38*, 201–221. https://doi.org/10.1016/j.dr.2015.07.008

Mavrikis, M., Geraniou, E., Gutierrez Santos, S., & Poulovassilis, A. (2019). Intelligent analysis and data visualisation for teacher assistance tools: The case of exploratory learning. *British Journal of Educational Technology*, *50*(6), 2920–2942. https://doi.org/10.1111/bjet.12876

Mavrikis, M., & Holmes, W. (2019). Intelligent learning environments: Design, usage and analytics for future schools. In *Shaping future schools with digital technology: An international handbook* (pp. 57–73). Springer.

Mavrikis, M., Rummel, N., Wiedmann, M., Loibl, K., & Holmes, W. (2022). Combining exploratory learning with structured practice educational technologies to foster both conceptual and procedural fractions knowledge. *Educational Technology Research and Development*, *70*(3), 691–712. https://doi.org/10.1007/s11423-022-10104-0

Meeter, M. (2021). Primary school mathematics during Covid-19: No evidence of learning gaps in adaptive practicing results. *Trends in Neuroscience and Education*, *25*, 100163. https://doi.org/10.1016/j.tine.2021.100163

Murnane, R. J., Willett, J. B., & Levy, F. (1995). The growing importance of cognitive skills in wage determination. *Review of Economics and Statistics*, *77*(2), 251–266.

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. U.S. Department of Education.

Ninaus, M., Cortez, R., Hazin, I., Kiili, K., Wortha, S. M., Klein, E., Weiss, E. M., & Moeller, K. (2023). The added value of game elements: Better training performance but comparable learning gains. *Educational Technology Research and Development*, *71*(5), 1917–1939. https://doi.org/10.1007/s11423-023-10263-8

Obersteiner, A., Van Dooren, W., Van Hoof, J., & Verschaffel, L. (2013). The natural number bias and magnitude representation in fraction comparison by expert mathematicians. *Learning and Instruction*, *28*, 64–72. https://doi.org/10.1016/j.learninstruc.2013.05.003

Obersteiner, A., Van Hoof, J., Verschaffel, L., & Van Dooren, W. (2015). Who can escape the natural number bias in rational number tasks? A study involving students and experts. *British Journal of Psychology*, *107*, 537–555. https://doi.org/10.1111/bjop.12161

Oliveira, G., Grenha Teixeira, J., Torres, A., & Morais, C. (2021). An exploratory study on the emergency remote education experience of higher education students and teachers during the COVID-19 pandemic. *British Journal of Educational Technology*, *52*(4), 1357–1376. https://doi.org/10.1111/bjet.13112

Parker, M., & Leinhardt, G. (1995). Percent: A privileged proportion. *Review of Educational Research*, *65*(4), 421–481.

Plass, J. L., Homer, B. D., & Kinzer, C. K. (2015). Foundations of game-based learning. *Educational Psychologist*, *50*(4), 258–283. https://doi.org/10.1080/00461520.2015.1122533

Pöhler, B., George, A. C., Prediger, S., & Weinert, H. (2021). Are word problems really more difficult for students with low language proficiency? Investigating percent items in different formats and types. *International Electronic Journal of Mathematics Education*, *12*(3), 667–687. https://doi.org/10.29333/iejme/641

Pöhler, B., & Prediger, S. (2015). Intertwining lexical and conceptual learning trajectories - A design research study on dual macro-scaffolding towards percentages. *Eurasia Journal of Mathematics, Science and Technology Education*, *11*(6), 1697–1722. https://doi.org/10.12973/eurasia.2015.1497a

Prediger, S., & Neugebauer, P. (2023). Can students with different language backgrounds profit equally from a language-responsive instructional approach for percentages? Differential effectiveness in a field trial. *Mathematical Thinking and Learning*, *25*(1), 2–22. https://doi.org/10.1080/10986065.2021.1919817

R Core Team. (2013). R: A language and environment for statistical computing. http://www.r-project.org/

Rau, M. A., Aleven, V., & Rummel, N. (2009). Intelligent tutoring systems with multiple representations and self-explanation prompts support learning of fractions. *Frontiers in Artificial Intelligence and Applications*, *200*(1), 441–448. https://doi.org/10.3233/978-1-60750-028-5-441

Rau, M. A., Aleven, V., & Rummel, N. (2016). Supporting students in making sense of connections and in becoming perceptually fluent in making connections among multiple graphical representations. *Journal of Educational Psychology*, *94*(4), 659. https://doi.org/10.1037//0022-0663.94.4.659

Rienties, B., Nguyen, Q., Holmes, W., & Reedy, K. (2017). A review of ten years of implementation and research in aligning learning design with learning analytics at the Open University UK. *Interaction Design and Architecture*, *33*, 134–154.

Ritchie, S. J., & Bates, T. C. (2015). Enduring links from childhood mathematics and reading achievement to adult socioeconomic status. *Psychological Science*, *24*(7), 1301–1308. https://doi.org/10.1177/0956797612466268

Ritter, S., Anderson, J. R., Koedinger, K. R., & Corbett, A. (2007). Cognitive tutor: Applied research in mathematics education. *Psychonomic Bulletin & Review*, *14*(2), 249–255.

Rittle-Johnson, B., & Koedinger, K. (2009). Iterating between lessons on concepts and procedures can improve mathematics knowledge. *British Journal of Educational Psychology*, *79*(3), 483–500. https://doi.org/10.1348/000709908X398106

Rittle-Johnson, B., & Koedinger, K. R. (2005). Designing knowledge scaffolds to support mathematical problem solving. *Cognition and Instruction*, *23*(3), 313–349. https://doi.org/10.1207/s1532690xci2303_1

Rivera-Batiz, F. L. (1992). Quantitative literacy and the likelihood of employment among young adults in the United States. *The Journal of Human Resources*, *27*(2), 313–328.

RStudio Team. (2015). *RStudio: Integrated development environment for R*. RStudio.

Salas-Pilco, S. Z., Yang, Y., & Zhang, Z. (2022). Student engagement in online learning in Latin American higher education during the COVID-19 pandemic: A systematic review. *British Journal of Educational Technology*, *53*(3), 593–619. https://doi.org/10.1111/bjet.13190

Schneider, M., & Siegler, R. S. (2010). Representations of the magnitudes of fractions. *Journal of Experimental Psychology: Human Perception and Performance*, *36*(5), 1227–1238. https://doi.org/10.1037/a0018170

Siegler, R. S., Duncan, G. J., Davis-kean, P. E., Duckworth, K., Claessens, A., Engel, M., Susperreguy, M. I., & Chen, M. (2012). Early predictors of high school mathematics achievement. *Psychological Science*, *23*(7), 691–697. https://doi.org/10.1177/0956797612440101

Siegler, R. S., Fazio, L. K., Bailey, D. H., & Zhou, X. (2013). Fractions: The new frontier for theories of numerical development. *Trends in Cognitive Sciences*, *17*(1), 13–19. https://doi.org/10.1016/j.tics.2012.11.004

Siegler, R. S., & Lortie-Forgues, H. (2015). Conceptual knowledge of fraction arithmetic. *Journal of Educational Psychology*, *107*(3), 909–918. https://doi.org/10.1037/edu0000025

Siegler, R. S., & Lortie-Forgues, H. (2017). Hard lessons: Why rational number arithmetic is so difficult for so many people. *Current Directions in Psychological Science*, *26*(4), 346–351. https://doi.org/10.1177/0963721417700129

Siegler, R. S., Thompson, C. A., & Schneider, M. (2011). An integrated theory of whole number and fractions development. *Cognitive Psychology*, *62*(4), 273–296. https://doi.org/10.1016/j.cogpsych.2011.03.001

Siegler, R. S., & Tian, J. (2022). Why do we have three rational number notations? The importance of percentages. In *Advances in child deveopment and behavior* (Vol. *63*, pp. 1–33). JAI. Elsevier.

Spitzer, M. W. H., Bardach, L., Strittmatter, Y., Meyer, J., & Moeller, K. (2024). Evaluating the content structure of intelligent tutor systems—A psychological network analysis. *Computers and Education Open*, *7*, 100198.

Spitzer, M. W. H., & Moeller, K. (2022). Predicting fraction and algebra achievements online: A large-scale longitudinal study using data from an online learning environment. *Journal of Computer Assisted Learning*, *38*, 1797–1806. https://doi.org/10.31234/osf.io/rw6b9

Spitzer, M. W. H., & Moeller, K. (2024). Performance increases in mathematics within an intelligent tutoring system during COVID-19 related school closures: A large-scale longitudinal evaluation. *Computers and Education Open*, *6*(January), 100162. https://doi.org/10.1016/j.caeo.2024.100162

Spitzer, M. W. H., Moeller, K., & Musslick, S. (2023). Assignment strategies modulate students' academic performance in an online learning environment during the first and second COVID-19 related school closures. *PLoS ONE*, *18*(5 May), 1–23. https://doi.org/10.1371/journal.pone.0284868

Stapel, M., Zheng, Z., & Pinkwart, N. (2016). An ensemble method to predict student performance in an online math learning environment. In *Proceedings of the 9th International Conference on Educational Data Mining* (pp. 231–238). ERIC.

Stigler, J., Givvin, K., & Thompson, B. (2010). What community college developmental mathematics students understand about mathematics. *MathAMATYC Educator*, *1*(3), 4–16.

St-Onge, C., Ouellet, K., Lakhal, S., Dubé, T., & Marceau, M. (2022). COVID-19 as the tipping point for integrating e-assessment in higher education practices. *British Journal of Educational Technology*, *53*(2), 349–366. https://doi.org/10.1111/bjet.13169

Tian, J., Braithwaite, D. W., & Siegler, R. S. (2020). How do people choose among rational number notations? *Cognitive Psychology*, *123*, 1–56.

Tian, J., & Siegler, R. S. (2017). Fractions learning in children with mathematics difficulties. *Journal of Learning Disabilities*, *50*(6), 614–620. https://doi.org/10.1177/0022219416662032

Tian, J., & Siegler, R. S. (2018). Which type of rational numbers should students learn first? *Educational Psychology Review*, *30*(2), 351–372. https://doi.org/10.1007/s10648-017-9417-3

Tomasik, M. J., Helbling, L. A., & Moser, U. (2020). Educational gains of in-person vs. distance learning in primary and secondary schools: A natural experiment during the COVID-19 pandemic school closures in Switzerland. *International Journal of Psychology*, *56*(4), 566–576. https://doi.org/10.1002/ijop.12728

Van Schoors, R., Elen, J., Raes, A., & Depaepe, F. (2021). An overview of 25 years of research on digital personalised learning in primary and secondary education: A systematic review of conceptual and methodological trends. *British Journal of Educational Technology*, *52*(5), 1798–1822. https://doi.org/10.1111/bjet.13148

VanLehn, K. (2011). The relative effectiveness of human tutoring, intelligent tutoring systems, and other tutoring systems. *Educational Psychologist*, *46*(4), 197–221. https://doi.org/10.1080/00461520.2011.611369

Vukovic, R. K., Fuchs, L. S., Geary, D. C., Jordan, N. C., Gersten, R., & Siegler, R. S. (2014). Sources of individual differences in children's understanding of fractions. *Child Development*, *85*(4), 1461–1476. https://doi.org/10.1111/cdev.12218

Whalen, K. A., Renkl, A., Eitel, A., & Glogger-Frey, I. (2024). Digital re-attributional feedback in high school mathematics education and its effect on motivation and achievement. *Journal of Computer Assisted Learning*, *40*(2), 478–493. https://doi.org/10.1111/jcal.12889

Wortha, S. M., Klein, E., Lambert, K., Dackermann, T., & Moeller, K. (2023). The relevance of basic numerical skills for fraction processing: Evidence from cross-sectional data. *PLoS ONE*, *18*(1 January), 1–15. https://doi.org/10.1371/journal.pone.0281241

Yan, L., Whitelock-Wainwright, A., Guan, Q., Wen, G., Gašević, D., & Chen, G. (2021). Students' experience of online learning during the COVID-19 pandemic: A province-wide survey study. *British Journal of Educational Technology*, *52*(5), 2038–2057. https://doi.org/10.1111/bjet.13102

SUPPORTING INFORMATION

Additional supporting information can be found online in the Supporting Information section at the end of this article.

How to cite this article: Spitzer, M. W. H., Ruiz-Garcia, M., & Moeller, K. (2025). Basic mathematical skills and fraction understanding predict percentage understanding: Evidence from an intelligent tutoring system. *British Journal of Educational Technology*, *56*, 1122–1147. https://doi.org/10.1111/bjet.13517