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A Psychological Network Analysis to Examine Interdependencies Between Fraction and Algebra Subtopics in an Intelligent Tutoring System

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ABSTRACT

Background: Many students face difficulties with algebra. At the same time, it has been observed that fraction understanding predicts achievements in algebra; hence, gaining a better understanding of how algebra understanding builds on fraction understanding is an important goal for research and educational practice.

Objectives: However, a wide range of algebra subtopics (e.g., *Using formulas* and *Simplifying products in formulas*) and fraction subtopics (e.g., *Adding and subtracting fractions*, *Multiplying and dividing fractions*) exist, and little is known about which specific fraction subtopics matter most for (i.e., best predict) which specific algebra subtopics. In addition to addressing across-topic subtopic correlations, a comprehensive understanding of within-topic subtopic correlations (i.e., among fraction subtopics and algebra topics, respectively) has not yet been achieved.

Methods: Here, we leveraged a large data set (3158 students; 257,321 problem sets) from an intelligent tutoring system (ITS) and employed state-of-the-art psychological network analysis to visualise and quantify interdependencies between students' performance on different fractions and algebra subtopics.

Results and Conclusions: We observed one robust correlation between a specific fraction and a specific algebra subtopic (*Fractions and the order of operations* and *Using formulas*). In addition, a larger number of within-topic subtopic correlations were observed. Importantly, cross-topic correlations and most within-topic correlations seemed to be driven by shared mathematical components (e.g., multiplication, operating rules or reading comprehension). Our findings advance the current understanding of mathematics learning and have implications for the design and improvement of ITSs, such as for developing automatic suggestions on which other subtopics to work on when a student encounters difficulties with a specific subtopic. Moreover, our study highlights the potential of psychological network analysis for analysing learning data from ITSs.

1 | Introduction

Over the past decade, numerous studies reported that fraction understanding is of utmost importance for learning mathematics

as it significantly predicts achievement on later, more advanced mathematical topics such as algebra (Bailey et al. 2012; Booth and Newton 2012; McMullen and Van Hoof 2020; Mou et al. 2016; Park and Esposito 2024; Siegler et al. 2012; Spitzer

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Summary

- What is currently known about this topic?
 - Fraction understanding significantly predicts later algebra performance.
 - Algebra serves as a gatekeeper for academic and professional success.
 - Intelligent tutoring systems (ITSs) collect fine-grained learning process data.
 - Psychological network analysis can reveal interdependencies between topics.
- What does this paper add?
 - Identifies Fractions and the order of operations as predicting using formulas.
 - Finds subtopics sharing mathematical components correlate more strongly.
 - Reveals stronger correlations within topics than across topics.
 - Applies psychological network analysis to process data from an ITS.
- Implications for practice/or policy
 - ITSs can suggest earlier subtopics when students struggle with a given subtopic.
 - Learning content can be structured based on shared mathematical components.
 - Developers should tag subtopics by skills like operating rules or multiplication.

and Moeller 2022; Torbeyns et al. 2015). Algebra achievement, in turn, was argued to serve as a gatekeeper for subsequent academic success in mathematics and science. In fact, poor algebra skills often preclude college acceptance and thus limit career options and access to better-paying jobs that require advanced mathematics skills (Murnane et al. 1995; National Mathematics

Advisory Panel 2008; Siegler et al. 2012). Accordingly, due to the link between fraction performance and later algebra performance, understanding fractions has been described as the ‘gatekeeper’s doorman’ (Booth and Newton 2012).

However, fractions are a complex topic that is typically broken down into different subtopics such as *Adding and subtracting fractions*, *Multiplying and dividing fractions* and *Fractions in context* (see Table 1). The same is the case for algebra with subtopics like *Using formulas*, *Simplifying products in formulas* and *Single brackets in formulas* (see Table 1). Importantly, associations between fraction and algebra understanding are well-established on the aggregate level (e.g., Bailey et al. 2012; Booth and Newton 2012; Mou et al. 2016; Siegler et al. 2012; Spitzer and Moeller 2022) and underpinned by theoretical considerations, such as the integrated theory of numerical development (e.g., Siegler and Lortie-Forgues 2014; Siegler et al. 2011). Nevertheless, it remains to be investigated which specific fraction subtopic(s) best predict(s) which specific algebra subtopic(s). For example, it is unclear which fraction subtopic best predicts understanding of the algebra subtopic *Using formulas*? Is it *Adding and subtracting fractions* or rather *Fractions in context*, or any other fraction subtopic? And in case a specific fraction subtopic is identified as the strongest predictor of students’ understanding of the algebra subtopic *Using formulas*, does this fraction subtopic also best predict performance in solving other (more elaborate) algebra problems, such as *Single brackets in formulas*? Or does another fraction subtopic best predict the algebra subtopic *Single brackets in formulas*? As these examples indicate, breaking down the known association of fraction and algebra understanding to their individual subtopics not only allows to tackle unsolved substantive questions but also poses an interesting methodological challenge: which one out of multiple predictor variables (fraction subtopics) presents the strongest relationship to which one out of multiple outcome variables (algebra subtopics)?

TABLE 1 | Abbreviated book-subtopic ID and average days passed since F1.

ID	Subtopic	Components	Days since F1
F1	Multiples and factors	Multiplication; Reading comprehension	0
F2	Prime numbers	Multiplication	3
F3	Forming and cancelling down fractions	Multiplication; Fraction magnitudes	5.75
F4	Fractions and decimal numbers	Fraction magnitudes	10
F5	Adding and subtracting fractions	Addition and subtraction; Calculating fractions	15
F6	Multiplying and dividing fractions	Multiplication and division; Calculating fractions	20
F7	Fractions in context exercises	Calculating fractions; Reading comprehension	24
F8	Fractions and the order of operations	Calculating fractions; Operating rules	26
F9	Fractions and powers	Calculating fractions	28
A1	Using formulas	Multiplication; Operating rules	42
A2	Setting up formulas	Operating rules; Reading comprehension	46
A3	Simplifying products	Multiplication; Operating rules	48
A4	Like terms	Addition and multiplication; Operating rules	53
A5	Single brackets	Operating rules	61

Importantly, more detailed insights into the pattern of interdependencies between algebra and fraction subtopics can feed back into the construction and development of the curriculum of learning content embedded within digital learning software. For example, software developers may organise the content in a way that, when students struggle with specific subtopics, earlier subtopics identified to predict understanding of the target subtopic may be automatically assigned to students for revision to ease their learning process for the target subtopic. Thus, guidance on which specific subtopics build upon each other is important.

Recent developments in psychological network analysis offer an ideal approach for easily addressing such questions, as they allow for visually representing and quantifying intricate interdependencies among various variables within a single easy-to-access illustration (Borsboom and Cramer 2013; Borsboom et al. 2021; Cramer et al. 2010; Epskamp et al. 2018; Spitzer, Bardach, et al. 2024). In particular, psychological network analysis with partial correlations allows for evaluating the contribution of a specific variable while controlling for the influence of other variables (Epskamp and Fried 2018). However, as the interdependencies between many different variables are considered, psychological network analysis requires large datasets to ensure sufficient statistical power (Epskamp and Fried 2018).

Recently, there has been a surge in the number of students worldwide learning mathematics through digital learning platforms, such as intelligent tutoring systems (ITSs), partly driven by the global impact of the COVID-19 pandemic (e.g., Meeter 2021; Spitzer et al. 2023; Tomasik et al. 2020). This presents an ideal opportunity to apply psychological network analysis, particularly because ITSs continuously collect vast amounts of process data as students learn (Koedinger et al. 2023; Ritter et al. 2007; Spitzer, Ruiz-Garcia, et al. 2025; Spitzer et al. 2023). In the present study, we seized this opportunity and evaluated a large dataset ($n = 3158$ students who worked on $n = 257,321$ problem sets) from an ITS for learning mathematics. The ITS provides a personalised learning environment in which students work on different fractions and algebra subtopics (see Section 3 for more details on the ITS). We used this rich dataset to evaluate which fraction subtopic(s) best predicted which algebra subtopic(s). We also addressed within-topic correlations (i.e., between fraction and algebra subtopics, respectively). Our study was guided by the assumption that shared basic mathematical components (e.g., whether two subtopics both require multiplication skills) may drive stronger correlations in both across-topic and within-topic performance.

2 | Scientific Background

2.1 | Fractions: The Gatekeeper's Doorman

The integrated theory of numerical development suggests that fraction understanding is a cornerstone for students when learning algebra (Siegler and Lortie-Forgues 2014; Siegler et al. 2012). This theory posits that mathematical concepts develop interdependently and scaffold upon one another, realising a hierarchy

in the development of mathematical skills and concepts (Siegler and Braithwaite 2017; Siegler and Lortie-Forgues 2014). In the context of algebra, fraction understanding serves as a foundational element, aiding in the transition to more complex algebraic concepts (Siegler et al. 2012). In particular, Siegler et al. (2012) argue that understanding fractions enhances problem-solving abilities, as many algebraic word problems involve fractions (e.g., $1/3 \times x = 2/3 \times y$, where y is two times as much as x). Thus, understanding the concept of fractions per se seems like a prerequisite for such formula problems.

The propositions of the integrated theory of numerical development are substantiated by longitudinal studies observing that understanding fractions is indeed important for learning algebra. In fact, it was repeatedly found that students with better fraction understanding also performed better on algebra later on (Bailey et al. 2012; Booth and Newton 2012; Siegler et al. 2012; Spitzer and Moeller 2022). For instance, Siegler et al. (2012) analysed data from more than 4000 students from the United Kingdom and the United States and observed that students' fraction understanding at the age of 10 years significantly predicted their performance in algebra as well as overall mathematical attainment 6 years later—even after controlling for whole number knowledge, reading skills, working memory, family income and general cognitive abilities. Another study with a sample of 212 students indicated that students' fraction understanding in sixth grade significantly predicted their mathematics achievement, including algebra, 1 year later in seventh grade (Bailey et al. 2012). Again, this result held when controlling for working memory, general cognitive ability, seventh-grade fractions comparison performance and sixth-grade overall mathematics achievement (Bailey et al. 2012; see also Mou et al. 2016 for a similar prediction from eighth to ninth grade). In summary, longitudinal evidence from studies with in-person testing provides converging evidence indicating that fraction understanding feeds into later algebra performance.

2.2 | Interdependencies Between Fractions and Algebra Observed in ITSs

ITSs have emerged as a tool to facilitate mathematical learning (Anderson et al. 1985, 1995; Khazanchi et al. 2024; Koedinger et al. 2023; Mavrikis et al. 2022; Ritter et al. 2007; Spitzer 2022; VanLehn 2011). Through their adaptive nature, ITSs offer the opportunity to scaffold tailored learning experiences and provide real-time feedback to students (Corbett et al. 2001; Koedinger et al. 2023; Ritter et al. 2007). In particular, the use of ITSs was found to facilitate mathematical learning by adapting learning content to the learner's needs, thereby optimising learning trajectories and outcomes (e.g., Rau et al. 2009; Rittle-Johnson and Koedinger 2009, 2005). Importantly, these systems log all interaction data while students engage with the learning materials. This process data can be evaluated to advance our understanding of numerical development and to improve these systems (e.g., Ritter et al. 2007). With respect to advancing current understanding of numerical development, studies using data from ITSs can complement and extend existing research on the relationships between fraction and algebra performance, which have largely relied on in-person testing. First, it is informative to

test whether findings obtained with in-person testing can be replicated using data obtained from ITSs. Second, due to the fine-grained data collected while large numbers of students work on multiple algebra and fraction subtopics over expanded periods of time within the ITSs, further highly relevant questions pertaining to the complex interdependencies between a range of different topics can be addressed.

For instance, recent work by Spitzer and Moeller (2022) used a large dataset of $n = 5294$ students who worked on $n = 1,050,897$ problem sets. They demonstrated that students' average algebra performance across different algebra subtopics was significantly predicted by average fraction performance across different fraction subtopics. Although this study contributes to the current understanding by replicating findings from previous longitudinal in-person assessments, it also raises the question of which specific fraction subtopics are most predictive of which specific algebra subtopics.

2.3 | Specifying Interdependencies Between Fraction and Algebra Subtopic

Siegler et al. (2012) argued that algebra problems involving fractions can only be solved by students who understand fractions (e.g., $1/3X = 2/3Y$, with X twice as large as Y) and thus, fraction understanding predicts algebra understanding. However, algebra problems may not only draw on fractions, but may also build on other mathematical concepts and procedures such as basic arithmetic (e.g., addition, subtraction, multiplication and division), operating rules (e.g., brackets before multiplication/division before addition/subtraction), but also topic-general skills such as reading comprehension (e.g., when working on word problems). For instance, working with formulas in algebra often requires mastery of basic arithmetic skills and understanding operating rules (e.g., $9 + 3 \times p - 9 \times q$ where $p = 4$ and $q = 5$). Students must also successfully apply these skills and rules when processing specific fraction problems (e.g., $9 + 3 \times 1/3 - 9 \times 1/2 = ?$). Therefore, we propose that students' performance on fraction subtopics that share such (mathematical) components with later algebra subtopics is more likely to be associated with performance on corresponding algebra subtopics sharing the same (mathematical) components. Moreover, we propose that the same principle should hold when looking at within-topic correlations. Specifically, relationships between specific fraction subtopics and specific algebra subtopics, respectively, should be stronger for subtopics building on the same mathematical component.

To the best of our knowledge, such fine-grained interdependencies between mathematical subtopics have not been investigated thus far. However, a better understanding of these interdependencies seems pivotal for understanding developmental trajectories of mathematical skills and, more practically, for optimising the design of ITS to better motivate revision suggestions. Specifically, by identifying the fraction subtopic(s) most predictive of later algebra subtopic(s), future iterations of ITSs can be refined to emphasise these critical subtopics, thereby improving the efficacy of mathematics education in digital environments. Importantly, psychological network analysis offers the possibility to evaluate such interdependencies between the fraction and algebra subtopics.

2.4 | Psychological Network Analysis

Psychological network analysis seeks to conceptualise behaviour as a multifaceted interplay of psychological and other components (Borsboom and Cramer 2013; Borsboom et al. 2021; Cramer et al. 2010; Epskamp et al. 2018). On a methodological level, psychological network analysis represents an advanced visualisation of a correlation matrix and is most suitable for large (i.e., many participants) and complex (i.e., many variables) datasets. In this case, psychological network analysis can help identify a possible framework for understanding how psychological and other components may interact by representing observed variables as nodes and the correlations between these variables as edges, with the width of these edges reflecting the strength of the association between the respective variables (Epskamp and Fried 2018; Epskamp et al. 2018).

The nodes of psychological networks can be illustrated using a force-directed layout, displaying nodes with strong correlations closer together and nodes with weaker correlations farther apart. This way, it is easy to identify clusters or communities of variables within the psychological network (Fruchterman and Reingold 1991).

In sum, the application of psychological network analysis is a data-driven approach to explore complex relationships between multiple observable variables by illustrating and quantifying their interdependencies. This allows one to reflect, comprehend and interpret complex patterns of interdependencies of observed variables. Due to this promising feature of psychological network analysis and its simple implementation within a few lines of code (see <https://osf.io/avq2d/>), this method has gained increasing attention recently. For example, psychological network analysis was successfully applied in a variety of disciplines, such as personality research (e.g., Costantini et al. 2015; Cramer et al. 2012), clinical psychology (e.g., Boschloo et al. 2015), social psychology (e.g., Dalege et al. 2016) and educational research (e.g., Bower et al. 2022; Malmberg et al. 2022; Spitzer, Bardach, et al. 2024; Spitzer, Ruiz-Garcia, et al. 2025; Tang et al. 2022; van Hoogmoed et al. 2024). However, to the best of our knowledge, there is a lack of research adopting psychological network analysis to investigate large data sets obtained from ITSs for learning mathematics to better understand complex patterns of interdependencies between mathematical (sub)topics. Accordingly, this study applied psychological network analysis to pursue questions about interdependencies and thus the prediction of the performance of algebra subtopic(s) based on fraction subtopic(s) implemented in an ITS.

2.5 | Study Overview

We applied psychological network analysis to a large dataset from an ITS to investigate which specific fraction subtopic(s) best predict(s) which specific algebra subtopic(s). In doing so, we also sought to gain insights into the potential of psychological network analysis as a powerful method to evaluate process data from ITSs in general that can benefit learning analytics research. In particular, we evaluated process data from students who used the ITS *bettermarks* in the Netherlands and

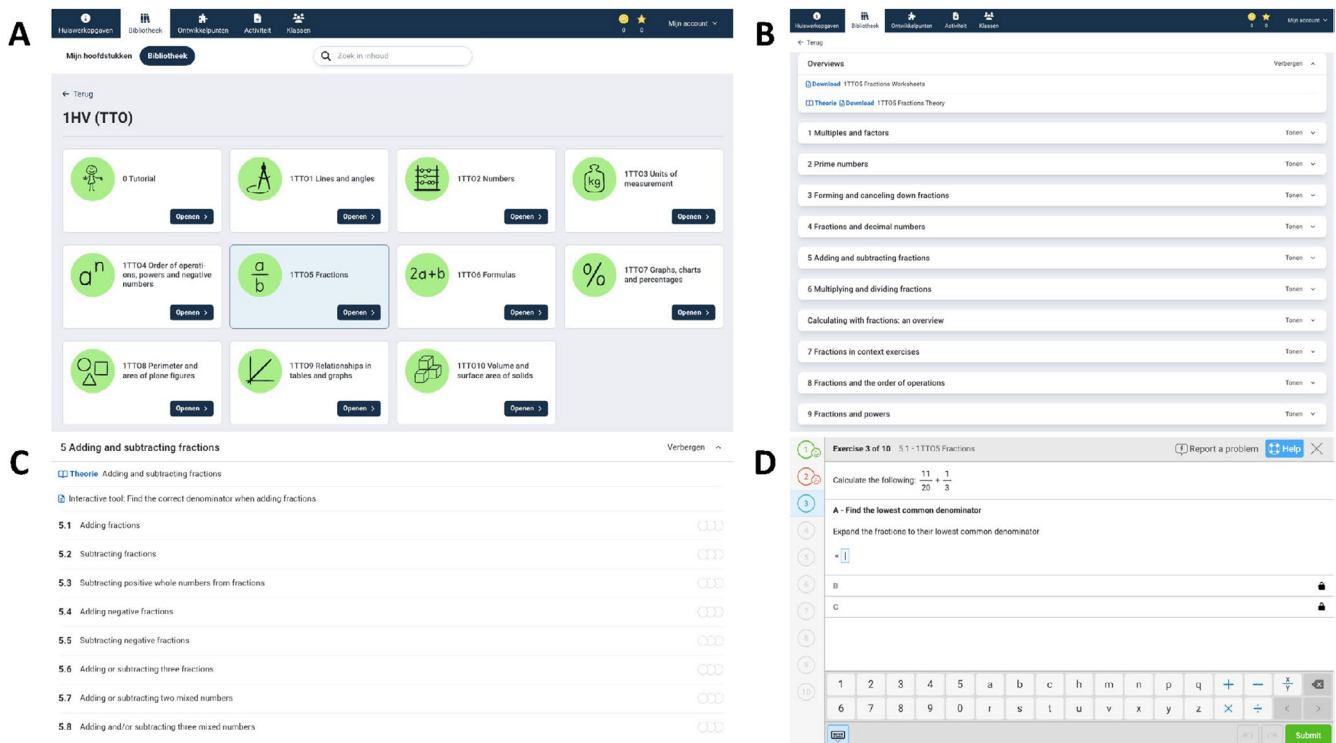


FIGURE 1 | The user interface and the content hierarchy of bettermarks. (A) Book selection within the bettermarks system used in the Netherlands. (B) Fraction subtopics. (C) Problem sets within the subtopic Adding and subtracting fractions. (D) Example of the third problem from the problem set: Adding fractions. This example problem comprises three procedural steps.

worked through fraction subtopics and algebra subtopics. The considerations described above propose that students' performance on fraction and algebra subtopics sharing the same mathematical components (e.g., drawing on the same operating rules) should correlate relatively highly. Against this background, we pursued the following research question: Which specific fraction subtopic(s) best predict(s) which specific algebra subtopic(s)?

In addition, we formulated the following hypotheses:

Hypothesis 1. *There are significant correlations between fraction and algebra subtopics (across-topic-correlations), whereby subtopics that share mathematical components (e.g., F8 & A1; sharing operating rules) should correlate more strongly than subtopics that do not share mathematical components.*

Hypothesis 2. *There are significant correlations among subtopics of the same topic (within-topic correlations), whereby subtopics that share mathematical components (e.g., F2 & F3; sharing multiplication) should correlate more strongly than subtopics that do not share mathematical components.*

3 | Methods

3.1 | The ITS Bettermarks

The ITS *bettermarks* facilitates learning mathematics through adaptive feedback and personalised instruction (e.g., Spitzer 2022; Spitzer, Bardach, et al. 2025; Spitzer, Ruiz-Garcia,

et al. 2025; Spitzer, Ruiz-Garcia, et al. 2024; Spitzer et al. 2023; Stapel et al. 2016; Whalen et al. 2024). *Bettermarks* covers curriculum-based topics (referred to as books within the ITS) for students to work through in class or at home. It is used in several countries, including the Netherlands, Germany and Uruguay (Spitzer 2022).

The content of *bettermarks* is structured hierarchically according to the respective mathematics curriculum. Different books cover different topics, which are further divided into subtopics called chapters in *bettermarks*. Each of the subtopics contains several problem sets (see Figure 1). Problem sets comprise, on average, eight mathematical problems.

In the Netherlands, *bettermarks* is typically used by schools that have digitised all their learning materials. Students in these schools systematically work through the topics and subtopics implemented in *bettermarks*, following the order in which these materials are structured. This systematic usage provides an ideal basis for scrutinising trajectories in mathematical learning, for instance, regarding the question of how performance on worked-through content (e.g., fraction problems) predicts performance on upcoming content (e.g., algebra). Thus, in this study, we only considered students who used *bettermarks* in the Netherlands.

The most frequent use case of *bettermarks* is that teachers assign problem sets to their students (around 90% of the worked-through problem sets in the Netherlands were assigned by teachers). Students may work on these assignments either at school or at home.

bettermarks incorporates several adaptive features. For instance, students and teachers receive immediate feedback on average performance scores (i.e., on accuracy; see Spitzer, Ruiz-Garcia, et al. 2025) for each assigned problem set. This enables them to monitor their progress on worked-through problem sets. In addition, when students commit specific errors, *bettermarks* recommends revising additional problem sets to close identified knowledge gaps. Students can repeat problem sets as many times as they wish; however, the parameterisation of problem sets is changed for each attempt. This modification serves the dual purpose of discouraging rote memorisation of previous results and encouraging active engagement with the problems.

In addition to receiving feedback upon completing entire problem sets, students are provided with immediate feedback after each individual problem, indicating whether it was solved correctly. In instances of incorrect solutions on single problems, students receive content-specific and error-contingent guidance, such as “find the common denominator first and then add the fractions,” when they erroneously attempted to add fractions with different denominators without first establishing a common denominator. This feedback mechanism aids in addressing specific errors and guides students toward the correct approach without explicitly providing them with the correct answer.

3.2 | Process Data From Bettermarks

Bettermarks logs the identification number (ID) of problem sets that users work through, as well as user ID, class ID and whether a problem set was assigned by a teacher or whether students self-selected the problem set. It also logs the ID of the book (i.e., fractions or algebra) as well as the respective subtopic (e.g., addition and subtraction of fractions, called chapters within *bettermarks*) to which the problem set belongs. Finally, *Bettermarks* logs the error rate on problem sets as well as the date and time when problem sets were computed.

Importantly, all data are completely anonymous, and it is impossible to trace back user data to any particular student using *bettermarks*. As all data are anonymous, no data on variables, such as age, gender or socioeconomic status, are available. The developers of *bettermarks* share their anonymous data for scientific purposes upon request. They had no role in the study design, and the reported results may not reflect their opinion.

3.3 | Inclusion Criteria for Data Analysis

We applied the following inclusion criteria for our data analysis. First, we only considered problems of two books used within the *bettermarks* system in the Netherlands: Fractions (abbreviated with F when listing subtopics) and Algebra (abbreviated with A when listing subtopics). These two books are usually worked through in Grade 6 (12–13 years of age). The data from the two books were also considered in another study; however, only data from students between January 2016 and December 2020 were used (Spitzer and Moeller 2022).¹ In the present study, we included all problem sets that were worked through between 1 January 2016 and 1 September 2023. The fraction book comprises nine subtopics, and the algebra book comprises five

subtopics (see Table 1). We only included problem sets assigned by teachers, as we assume that teachers only assign problem sets suited for their students' age range.

Another inclusion criterion was that we only included students who worked through at least five fraction and algebra problem sets per subtopic (see Figure 5 for descriptive variables). Additionally, we only considered algebra problem sets that were completed after fraction problem sets to ensure the directionality of our results (fraction performance predicting algebra performance). Moreover, we checked whether students worked through subtopics by subtopic by looking into the average days passed between the first subtopic of the fraction book (F1) students completed and each of the respective following subtopics (see “Days since F1” in Table 1). For instance, the second subtopic of the fraction book F2: *Prime numbers* was worked through 3 days after students completed the first subtopic of the fraction book F1: *Multiples and factors*. In contrast, the last subtopic of the algebra book A5 *Single brackets* was completed 60 days after students worked through F1. In case students repeated a problem set (average repetition rate: 1.57 repetitions per problem set; also see Figure 5 for repetition rate distribution), we only considered their best result as a proxy for their performance. Please also see the Supporting Information S1 for results on students' first attempt, which did not differ substantially from the results of their best attempt.

Finally, we only included students who worked through each subtopic. Based on these inclusion criteria, our final sample comprised 3158 students (from 246 different classes) who worked on 121 different problem sets and a total of 257,321 problem sets.

3.4 | Included Subtopics

We considered the following subtopics of the fractions and algebra book. The first four fraction subtopics, F1–F4, deal with the acquisition of fundamental skills that are crucial for comprehending and manipulating fractions. For each subtopic, exemplary problem sets are provided in Figure 2. In particular, F1 (Multiples and factors) encompasses a series of problems that require students to identify common multiples or the greatest common factors (requiring multiplication skills), in addition to word problems (requiring reading comprehension skills). Building on this, F2 (Prime numbers) comprises problem sets on how to discover prime numbers under particular circumstances, as well as how to express numbers as products of prime factors, and therefore also requires multiplication skills. Students investigate constructing and simplifying fractions in subtopic F3: *Forming and cancelling down fractions*, where they work on recognising, expressing, converting and simplifying improper and mixed fractions using prime and common factors (requiring multiplication skills and understanding of fraction magnitudes). Subtopic F4: *Fractions and decimal numbers* proceeds with a bidirectional conversion of decimals, mixed numbers and fractions, with an additional focus on simplifying fractions (requiring understanding fraction magnitudes).

The next five fraction subtopics (F5–F9) are illustrated in Figure 3 and entail the application of fundamental arithmetic principles and the accurate sequencing of arithmetic operations

A

Opgave 1 van 12 1.3 - 1TT05 Fractions

The multiples of 3 are: 3, 6, 9, 12, 15, ...
The multiples of 4 are: 4, 8, 12, 16, 20, ...
Find all of the common [multiples](#) of 3 and 4 that are less than 17.

A - Find multiples

Insert any extra boxes you need:

B

Opgave 1 van 7 2.2 - 1TT05 Fractions

Express 22 as the [product](#) of two [prime numbers](#).

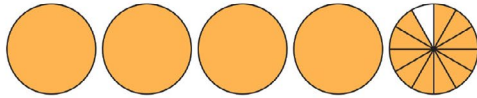
A

22 =

C

Opgave 1 van 9 3.6 - 1TT05 Fractions

Which of the following [mixed numbers](#) is shown in the circles?



A - Give the mixed number

Select from:

☐ $12\frac{11}{4}$ ☐ $4\frac{12}{11}$ ☐ $11\frac{4}{12}$ ☐ $4\frac{11}{12}$

D

Opgave 1 van 8 4.2 - 1TT05 Fractions

Write the following decimal number as a [fraction](#).

A - Write as a fraction

Write the fraction in simplest form.

0.7 =

FIGURE 2 | Exemplary problem sets of the first four fraction subtopics. (A) In subtopic F1 (Multiples and factors), the focus is on finding common multiples of two given numbers. (B) Drawing from subtopic F2 (Prime numbers), this problem involves expressing a number as the product of two prime numbers. (C) Based on F3 (Forming and cancelling down fractions), this problem requires selecting the mixed number that corresponds to the visual representation provided. (D) The solution to this problem, as part of subtopic F4 (Fractions and decimal numbers), is to convert a decimal number into a fraction in its simplest form.

A

Calculate the sum:
 $\frac{1}{7} + \frac{1}{6} + \frac{1}{3}$

=

Steps:

A - Finding the lowest common denominator

B - Adding fractions with a common denominator

C - Cancelling a fraction down to lowest terms

B

Calculate: $\frac{4}{5} \div \frac{7}{8}$

=

Steps:

A - Finding the reciprocal value

B - Multiplying fractions

C

Justin is shopping for his grandfather.
He buys $3\frac{1}{2}$ kg of apples, $2\frac{1}{4}$ kg of bananas and 250 g of grapes.
How many kilos of fruit does Justin buy in total?



Justin buys a total of kg of fruit.

D

Opgave 1 van 6 8.4 - 1TT05 Fractions

Calculate the following:

A - Follow the correct order of operations

$\frac{7}{9} \times \left(\frac{1}{7} - \frac{5}{7} \right) =$

E

Opgave 1 van 6 9.1 - 1TT05 Fractions

Write the following as a [power](#).

A - Collect the factors to form a power

$\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} =$

FIGURE 3 | Exemplary problem sets of the latter five fraction subtopics. (A) In F5 (Adding and subtracting fractions), the exemplary problem requires calculating the sum of three fractions by finding a common denominator and simplifying the result. (B) Subtopic F6 (Multiplying and dividing fractions) focuses on multiplying fractions, requiring the reciprocal to be found before performing the multiplication. (C) Drawing from F7 (Fractions in context exercises), this problem presents a real-world scenario in which various fractions must be combined to determine a total weight. (D) The example from subtopic F8 (Fractions and the order of operations) emphasises the relevance of the correct sequence of operations, involving brackets, multiple fractions and negative values. (E) An exemplary problem from subtopic F9 (Fractions and Powers) involving raising fractions to a power.

involving fractions (i.e., they all require the mathematical component calculating fractions). The subtopic F5: *Adding and subtracting fractions* is on the addition and subtraction of fractions. It involves understanding how to handle different types of fractions, including whole numbers, negative fractions and mixed numbers. The subsequent subtopic is on multiplying and dividing fractions, as outlined in F6: *Multiplying and dividing fractions*. Subtopics F5 and F6 also comprise the conversion of mixed fractions and the cancellation of fractions, where feasible. Furthermore, subtopic F7: *Fractions in context exercises* introduces more challenging word problems that apply all previously practised arithmetic operations within practical scenarios (requiring reading comprehension skills in addition to calculating with fractions). Building on these skills, subtopic F8: *Fractions and the order of operations* presents more complex tasks requiring attention to the correct order of operations, including calculations involving multiple fractions, brackets and negative values (requiring calculating fractions skills and knowledge on operating rules). Finally, subtopic F9: *Fractions and powers*

extends these concepts by introducing exponent rules applied to fractions, covering tasks such as raising fractions to powers and simplifying the resulting expressions (requiring calculating fractions skills).

With respect to algebra (for exemplary problems see Figure 4), subtopic A1: *Using formulas* introduces substituting values into formulas and calculating the resulting expressions while applying the distributive law and following the correct order of operations (requiring operating rules). The following subtopic, A2: *Setting up formulas*, requires the reverse process of putting together algebraic formulas from word or real-world problems by identifying relationships between variables (requiring operating rules and reading comprehension skills). Subtopic A3: *Simplifying products* focuses on simplifying algebraic products by combining products (requiring multiplication skills and operating rules). In subtopic A4: *Like terms*, problems involve identifying and combining like terms, with a similar goal of further simplifying expressions (requiring addition and multiplication skills as well as

A

Calculate the value of the expression

$$7(-2-3m)(8-3n)-2$$

for $m=0$ and $n=4$.

$$7(-2-3 \times \square)(8-3 \times \square)-2$$

Steps:

- A - Substituting values in expressions
- B - Calculate the brackets in expressions
- C - Performing multiplication in expressions
- D - Calculation of the expression value

B

Opgave 1 van 7 2.1 - 1TTO6 Formulas Problem / feedback Hulp

The Martinez family wants to rent a car for their vacation. They have to pay \$70 per day. They also have to pay a one-time fee of \$30.

Follow the steps below to calculate how much it costs to rent the car for 15 days.

A - Set up the formula

Set up the formula for the total cost of renting for a number of days.

=×+

number of days
cost
price per day
30
70
100

C

Opgave 1 van 6 3.2 - 1TTO6 Formulas Problem

Match the simplified expression to its correct unsimplified form.

A - Simplify as much as possible

$18 \times b \times 13 \times m =$

$13 \times q \times 12 \times p =$

$10 \times a \times 11 \times t =$

$10 \times m \times 10 \times x =$

100mx

234bm

156pq

110at

D

Opgave 1 van 7 4.8 - 1TTO6 Formulas

Simplify the expression as much as possible.

A - Collect the like terms

$$5p + 14 + 6q + 3 + 7q = \square$$

E

Simplify $(-4c^2 + 11c) + (17c - 7c^2)$.

Simplify the expression so that it does not contain brackets.

- ☐ $-4c^2 + 11c - 17c + 7c^2$
- ☐ $-4c^2 + 11c - 17c - 7c^2$
- ☐ $4c^2 + 11c + 17c - 7c^2$
- ☐ $-4c^2 + 11c + 17c - 7c^2$

Steps:

- A - Expanding the brackets in a sum of algebraic expressions
- B - Collecting the like terms in an algebraic expressions

FIGURE 4 | Exemplary problems of Algebra subtopics. (A) Problems in subtopic A1 (Using formulas) focus on calculating the value of an expression by substituting values for variables and following the correct order of operations. (B) Subtopic A2 (Setting up formulas) involves problems that require setting up a formula to calculate costs for a rental scenario and identify relationships between variables. (C) For subtopic A3 (Simplifying products), the exemplary problem illustrates multiplication of terms and constants to achieve the correct simplified expression. (D) Problems in subtopic A4 (Like terms) involve simplifying an expression by combining like terms. (E) In subtopic A5 (Single brackets), problems require expanding brackets and combining like terms, resulting in a simplified expression without brackets.

requiring operating rules). In the last algebra subtopic A5: *Single brackets*, building on previous skills, the focus shifts to more advanced expanding and simplifying bracketed expressions, involving numbers, variables, powers, negatives and combining like terms (requiring operating rules). Critically, none of the algebra problems across all subtopics included in the algebra topic comprises problems involving fractions. Thus, all problems within the algebra book could be solved without understanding fractions.

3.5 | Data Analysis

Data analysis was conducted using R (R Core Team 2013). We computed the average error rate for each student on each subtopic and normalised error rates by applying a z-transformation for our psychological network analysis. This data table served as the basis for running the psychological network analysis.

Before conducting the psychological network analysis and moving to our main research questions, we computed students' average performance in fractions and algebra at the topic level rather than the subtopic level, and looked at the correlation between these two aggregate measures. This served as a check to examine whether the correlation between overall fraction and algebra performance that has been repeatedly observed in previous studies also emerges in our data set.

We used the R packages qgraph with the EBICglasso method to conduct the network analysis (Epskamp et al. 2012) and the bootnet package for bootstrapping analysis (Epskamp et al. 2018). EBICglasso estimates sparse and interpretable psychological networks by shrinking small partial correlations toward zero (via graphical LASSO) and selecting the optimal model based on a complexity-penalised fit criterion (via EBIC). This approach retains only the most robust relationships between variables, reducing noise and overfitting, and is particularly well suited for high-dimensional data.

Nodes in the resulting network plots represent fraction subtopics (coloured in blue ink) or algebra subtopics (coloured in red ink). Edge width represents the strength of the partial correlation, with thicker edges reflecting larger partial correlations. The weights of the edges connecting nodes (representing correlation strength) are estimated, and the accuracy of these estimates increases with increasing sample size (Epskamp and Fried 2018). We applied a threshold to only consider partial correlations of $r=0.1$ or higher, as we were only interested in correlation coefficients representing at least small effect sizes. In addition, the 95% confidence intervals (see description below) of correlations below $r=0.1$ may overlap with 0, challenging the robustness of correlation estimates. We, therefore, selected a threshold of $r=0.1$. We considered this threshold before data analysis. We further provided the actual partial correlation as labels on the edges. In addition, we applied the Fruchterman–Reingold algorithm to our network layout, which places nodes that highly correlate with each other closer together, whereas nodes that correlate less are presented further apart (Fruchterman and Reingold 1991). This aligns well with our goal of facilitating intuitive visual interpretation of the network structure. Other layouts, such as the spring layout, do not arrange variables according to their correlations and were therefore not applied.

Finally, we also ran a bootstrapping procedure to evaluate the robustness of the estimated edges and whether they differed significantly from zero. The bootstrapping procedure estimated the 95% confidence intervals (95% CIs) for each edge, allowing us to quantify whether each edge significantly differed from zero. It also allowed us to compare the difference in correlation strength between correlations. In other words, the first bootstrapping procedure allowed us to estimate the robustness of a correlation against zero, and the second bootstrapping procedure allowed us to quantify whether each correlation is significantly larger or smaller compared to each of the other possible correlations. This allowed us to evaluate relative differences between correlations (for in-depth details on controlling for alpha inflation, see Epskamp and Fried 2018).

4 | Results

Figure 5 shows descriptive statistics. In general, the Pearson correlation between the fraction topic and the algebra topic was $r=0.74$, with $p<0.001$, suggesting a strong correlation at the aggregate level. The following psychological network analysis evaluated associations between fractions and algebra subtopics.

The psychological network analysis is shown in Figure 6, and the results of the bootstrapping analysis are depicted in Figures 7 and 8. Table 1 lists all subtopics in the fraction (F) and algebra (A) books, including the average days that passed between the completion of the first subtopic of the fraction book by the students and each subsequent subtopic in the two books. It indicates that students computed on average each subtopic in the order of their appearance and that teachers tended to stick to the order in which the subtopics were presented within the ITS and assigned (all worked-through problems were assigned to students by their teachers). Finally, we further substantiated the analysis by providing centrality measures, such as strength, closeness and betweenness in Figure S6.

4.1 | Descriptive Results

The descriptive results indicated that the students computed more than 80 problem sets and at least 40 problem sets, on average (see Figure 5A). Repetitions of problem sets were rare, with an average repetition rate of 1.5 problem sets per student (see Figure 5B). For each subtopic, over 5000 problem sets were worked on (see Figure 5C). Furthermore, the average error rate for each subtopic is given in Figure 5D. We observed considerable variance in error rates between subtopics, with the highest error rate of 50% for the subtopic F7 *Fractions in context exercises*, which comprises fraction word problems, and the lowest error rate of 2% for the subtopic F4 *Fractions and decimal numbers*.

4.2 | Psychological Network Analysis

With respect to our research question ‘Which specific fraction subtopic(s) best predict(s) which specific algebra subtopic(s)?’, our results on students' best attempt revealed one partial correlation of $r=0.12$ between the fraction subtopic F8 *Fractions*

Algebra

- A1: Using formulas
- A2: Setting up formulas
- A3: Simplifying products
- A4: Like terms
- A5: Single brackets

Fractions

- F1: Multiples and factors
- F2: Prime numbers
- F3: Forming and canceling down fractions
- F4: Fractions and decimal numbers
- F5: Adding and subtracting fractions
- F6: Multiplying and dividing fractions
- F7: Fractions in context exercises
- F8: Fractions and the order of operations
- F9: Fractions and powers

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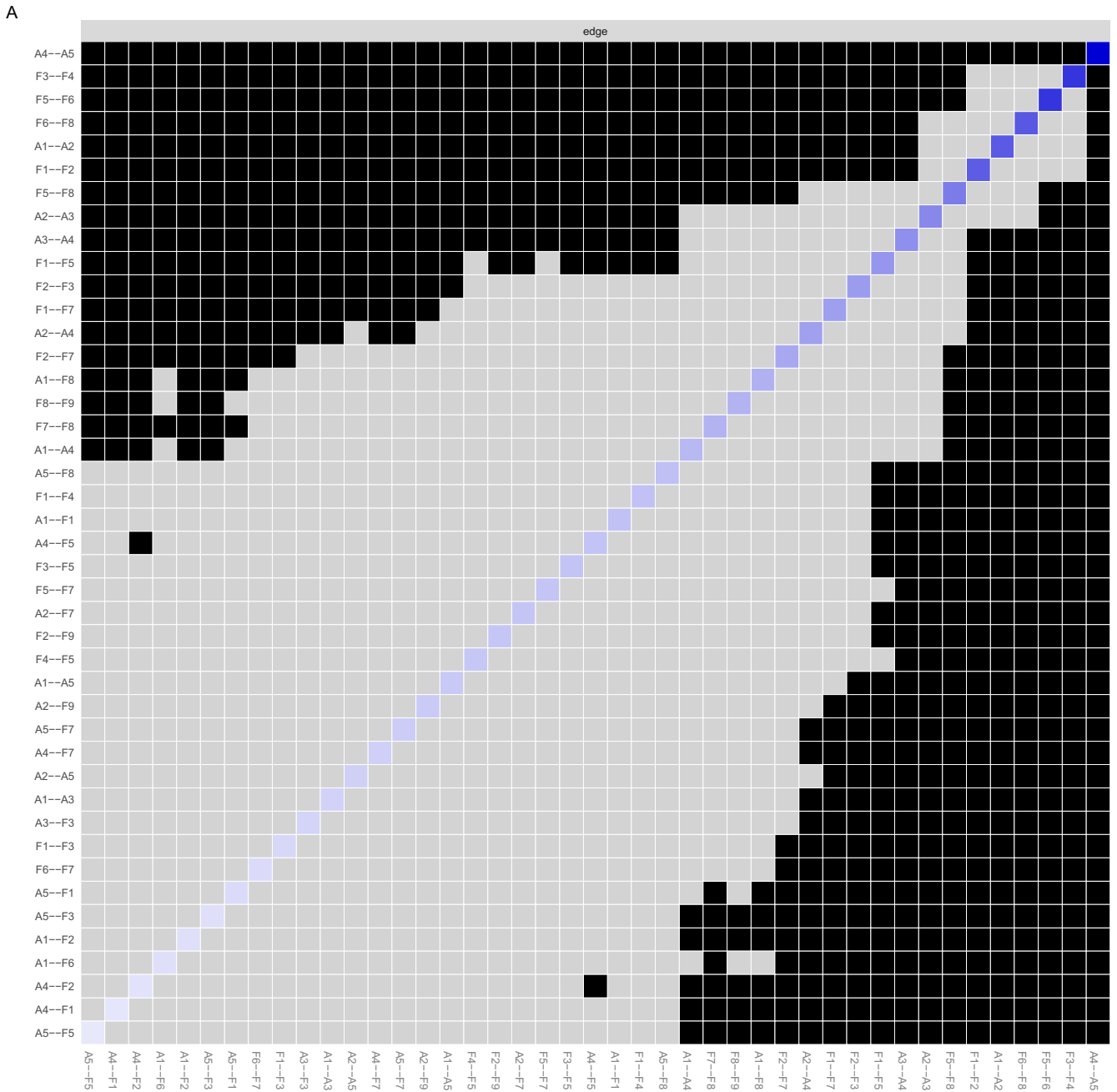


FIGURE 8 | Relative difference in correlation strength. Black squares indicate a significant difference between one partial correlation and another partial correlation, while grey squares indicate no significant difference between two partial correlations.

and the order of operations and the algebra subtopic A1 *Using formulas* (see Figure 6). Importantly, the bootstrapping procedure indicated that this partial correlation was significantly different from zero (see Figure 7) and was also significantly higher compared to the other correlations between fraction subtopics and the first algebra subtopic A1 *Using formulas* (except the subtopic F1 *Multiples and factors* and F6 *Multiplying and dividing fractions*; see Figure 8). The two subtopics F8 and A1 both share the same mathematical component (operating rules; see Table 1). This finding therefore provides support for our hypothesis that fraction subtopics sharing the same mathematical components as algebra subtopics should be positively associated.

Our psychological network analysis results also indicated that subtopics of the same topic that share mathematical components correlate relatively highly. In particular, we observed that the first two fraction subtopics F1 and F2, sharing the multiplication component, correlated relatively highly ($r=0.25$) and F2 and F3, which also shared the multiplication component, also correlated relatively highly ($r=0.15$, compared to the applied threshold of 0.1 and correlations between other variables assessed in this study). F3 was further associated with F4, and both subtopics shared the fraction magnitude understanding component ($r=0.31$). Furthermore, the three fraction subtopics F5, F6 and F8 all shared the same component of calculating fractions, and they were all relatively highly correlated with each other (F5 and

F6: $r=0.31$; F5 and F8: $r=0.20$; F6 and F8: $r=0.26$), while F9, which was on calculating with powers, was only associated with F8 ($r=0.12$). Finally, F7, which required reading comprehension skills when calculating fractions, was connected with F1 (which shared the reading comprehension component; $r=0.15$) and F8 (which shared the calculating fractions component; $r=0.12$). F2 also correlated with F7, but both subtopics shared no components ($r=0.14$). We also observed a partial correlation of $r=0.16$ between F1, which was on multiplication and knowledge of factors, and F5, which was on adding and subtracting fractions—sharing no components.

Considering correlations between algebra subtopics, we observed relatively high correlations between A1 and A2 ($r=0.26$) and A1 and A4 ($r=0.11$). All three subtopics required students to successfully apply operating rules. In addition, A1 and A4 also require multiplication skills. We also observed a relatively high correlation between A2 and A3 ($r=0.18$) as well as between A2 and A4 ($r=0.15$). Again, A2, A3 and A4 required students to successfully apply operating rules. In addition, A3 correlated relatively high with A4 ($r=0.17$), and both subtopics also share the multiplication component. Finally, A4 and A5 correlated relatively high ($r=0.40$), and both shared the operating rules component.

In sum, the results of the psychological network analysis indicated that fraction subtopics clustered together and algebra subtopics clustered together, as indicated by edges primarily connecting nodes among fraction and algebra subtopics, respectively. Furthermore, we observed relatively high partial correlations between subtopics that shared similar mathematical components.

5 | Discussion

We applied psychological network analysis to a large dataset obtained from an ITS to evaluate predictions of specific fraction subtopics on specific algebra subtopics. This provided a more differentiated picture of relations between algebra and fraction performance and thus expands on previous results indicating an association between these two topics on the aggregate level (Bailey et al. 2012; Barbieri et al. 2021; Booth and Newton 2012; McMullen and Van Hoof 2020; Mou et al. 2016; Park and Esposito 2024; Siegler et al. 2012; Spitzer and Moeller 2022; Torbeyns et al. 2015). In addition to investigating across-topic associations, we also examined within-topic associations to contribute to a comprehensive understanding of mathematics learning. Our study was guided by the assumption that both across- and within-topic correlations should be more pronounced for subtopics which share similar basic mathematical components (e.g., multiplication or operating rules).

With regard to across-topic correlations, the results from our psychological network analysis revealed a significant association between the fraction subtopic F8 *Fractions and the order of operations* and the algebra subtopic A1 *Using formulas*. Thus, we were able to identify a specific fraction subtopic that statistically significantly predicted a specific algebra subtopic while controlling for the influence of all other subtopics within the psychological network analysis.

In addition, the strength of the partial correlation between the fraction subtopic F8 *Fractions and the order of operations* and the algebra subtopic A1 *Using formulas* was significantly larger compared to the correlation strength between most of the other fraction subtopics and the first algebra subtopic A1 *Using formulas* (except the subtopic F1 *Multiples and factors* and F6 *Multiplying and dividing fractions*; see Figure 8). This indicates that the fraction subtopic F8 *Fractions and the order of operations* seem to include important learning content that helps students to master the first algebra subtopic A1 *Using formulas*. Overall, this finding highlights the value of investigating correlations between fraction and algebra subtopics, as this approach enables a more fine-grained understanding of the importance of specific subtopics—an insight that cannot be captured by examining correlations at the aggregate level.

Our study also sheds light on within-topic correlations. We obtained a number of significant associations between specific fraction subtopics and specific algebra subtopics, respectively. These within-topic correlations underline the interdependencies between subtopics within each topic. Given that within-topic subtopics all address the same overarching mathematical topic (i.e., fractions or algebra), the greater number of effects observed for within-topic correlations compared to across-topic correlations is conceptually consistent and supports the validity of the structure within the ITSs (for a similar finding, also see Spitzer, Bardach, et al. 2024).

While documenting the existence of relationships between specific subtopics, both across and within topics, adds to the current understanding, it does not yet offer insights into the mechanisms driving these associations. Specifically, what factors might explain why some subtopics exhibit significant associations with each other, while others do not? Our study largely supported the hypothesis that subtopics including the same mathematical component (e.g., multiplication) should be correlated. This held both for the one significant across-topic correlation and for most of the within-topic correlations. This finding advances knowledge of mathematical learning and highlights the importance of basic mathematical operations in fostering stronger connections between specific subtopics.

These findings align with and extend existing research emphasizing the foundational role of basic mathematical skills and concepts for later mathematical achievement. In particular, previous studies indicated that proficiency in basic arithmetic operations, such as multiplication and division, significantly predicts students' ability to learn more complex mathematical concepts, including fractions and algebra (e.g., Bailey et al. 2014; Siegler et al. 2012). By demonstrating that subtopics sharing the same underlying mathematical components are correlated more strongly, our results provide further empirical support for the idea that conceptual coherence and transferability of procedural knowledge are critical mechanisms in mathematical learning. This reinforces theoretical frameworks—such as the integrated theory of numerical development—proposing that the development of advanced mathematical competencies is scaffolded by mastery of more fundamental operations (Bailey et al. 2012; Libertus et al. 2011; Rittle-Johnson et al. 2001; Siegler et al. 2011) and highlights the importance of explicitly structuring curricula

to build on these shared components (National Mathematics Advisory Panel 2008).

Interestingly, we additionally observed that subtopics sharing the same components correlated most strongly if they were sequential. For example, F8 shared the same components with A1 but also with the other algebra subtopics. This is likely due to the fact that all algebra topics require operating rules, which are needed to process formulas. However, the strongest association was found between F8 and A1, whereas the correlations between F8 and the other algebra subtopics were not substantial. Considering the strong correlation between A1 and the other algebra subtopics as well as the relatively strongest cross-topic correlation between F8 and A1, we propose that A1 subsumes the variance explained by F8 for all other subtopics built on A1. This pattern also aligns with theoretical accounts emphasising the hierarchical and cumulative structure of mathematical learning (Bailey et al. 2012; Rittle-Johnson et al. 2001; Siegler et al. 2011). Taken together, our findings underscore the importance of both component similarity and sequential structure in shaping mathematics learning trajectories.

5.1 | Implications

Importantly, our findings also have practical implications for ITS developers. In particular, the results of our psychological network analysis suggest that when students struggle with a given subtopic, they may revisit specific previous subtopics and the one they struggle with builds on most strongly. Hence, we recommend implementing automatic dynamic sequencing based on a model that considers interdependencies between implemented subtopics to identify which prior subtopics correlate most strongly with the subtopic students struggle with. For instance, when a student struggles with the algebra subtopic A1 (*Using formulas*), the system should automatically prompt the student to revisit fraction subtopic F8 (*Fractions and the order of operations*), which our analysis revealed as a strong predictor of success in A1. More generally, this approach would allow ITSs to dynamically map learning trajectories by leveraging meaningful associations between subtopics. By integrating a psychological network analysis model into the ITS architecture, developers may well create systems that (i) can diagnose student difficulties in real time and (ii) deliver personalised, data-driven recommendations for additional learning materials for students to work through to facilitate further learning. However, more research is needed to develop such systems and evaluate whether such dynamic and adaptive sequencing of mathematical subtopics based on their observed interdependencies would indeed lead to better learning outcomes.

In addition, our results emphasise the need for ITS developers to track fine-grained and content-specific data, such as specific mathematical components that learning materials comprise (e.g., requiring reading comprehension skills). In particular, our findings indicate that such data allow for a better understanding of observed interdependencies between subtopics beyond the mathematical content itself. For example, when students struggle with the subtopic A1 (*Using formulas*), they may have difficulties understanding formulas in general, but they may

also struggle with procedures such as the order of operations that they may not have fully understood in previous topics. As such, revisiting and mastering previous learning materials on fractions that also address the order of operations, such as F8 (*Fractions and the order of operations*), may help these students to overcome their difficulties working on an already learnt and thus more familiar topic (i.e., fractions). This, in turn, may ultimately help students master specific algebra subtopics that require students to apply operating rules.

From a substantive-methodological perspective, our findings point toward the considerable potential of using psychological network analysis to advance research on understanding students' engagement with ITS in general and research on mathematics learning in particular. Psychological network analysis allows for quantifying process data with easy-to-access visuals that reflect the quantitative relationships between variables (Epskamp and Fried 2018; Epskamp et al. 2018). As such, psychological network analysis seems well suited to reduce large amounts of process data from ITS to identify meaningful and interpretable patterns in the data and to address substantive questions on learning processes within ITSs.

5.2 | Limitations and Directions for Future Research

Although meaningful, there are nevertheless aspects to consider when interpreting the results of the current study. The current study builds upon a rich data set of process data. However, due to the anonymisation of the data, we could not investigate the role of individual differences with respect to learner characteristics, as no participant-specific information was available. Therefore, future research may expand on our study by considering individual learner characteristics such as motivation, personality, cognitive abilities, prior academic achievement, socioeconomic status, or math anxiety (e.g., Bardach et al. 2023; Meyer et al. 2023) and their effects on learning trajectories within ITSs (Hilz et al. 2023a, 2023b). In addition, it would be illuminating to combine the process learning data used in our study with students' self-reported learning behaviour and emotional dynamics during learning, as well as other behavioural assessments (e.g., emotion recognition systems, eye-tracking, heart rate variability and EEG data) to dig even deeper into the details of mathematics learning with ITSs.

Another limitation may be that we used specific subtopics integrated within *bettermarks*, and other ITSs may have different subtopics. However, the general approach of applying psychological network analysis should be transferable to and informative on the structure and associations of (sub)topics in other ITSs—for learning mathematics, but also other content topics with (sub)topics building on each other. We only included students who worked through fraction problem sets before working through algebra problems to ensure the directionality of our results (i.e., fraction performance predicts algebra performance). However, we did not control for sequencing between the topics. Therefore, the relatively high correlations between fraction and algebra subtopics, respectively, could partially stem from time effects as subtopics that are closer together in time might correlate more due to time proximity, not because of feature

proximity. However, we believe that the limitation of not controlling for every possible sequence has to be viewed in light of a trade-off between internal and external validity (evaluating real-world data) and a large dataset.

5.3 | Conclusions

Taken together, the current study provides a first application of psychological network analysis with partial correlations to process data from an ITS. In particular, we observed relatively high partial within-topic correlations among fractions as well as among algebra subtopics. In addition, one fraction subtopic (F8: *Fractions and the order of operations*) was identified to best predict another algebra subtopic (A1: *Using formulas*). We furthermore demonstrated that shared mathematical components (e.g., operating rules) may largely drive within-topic and across-topic correlations. In sum, these results highlight the potential of psychological network analysis for identifying patterns of associations within subtopics of the same overarching topic as well as between subtopics of different topics and for advancing our understanding of mathematics learning.

Author Contributions

Markus W. H. Spitzer: conceptualization, methodology, writing – original draft, formal analysis, visualization, data curation, supervision. **Lisa Bardach:** conceptualization, writing – original draft, supervision. **Eileen Richter:** writing – review and editing. **Younes Strittmatter:** writing – review and editing, conceptualization. **Korbinian Moeller:** writing – original draft, conceptualization, supervision.

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Conflicts of Interest

The authors declare no conflicts of interest.

Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Endnotes

¹ This study investigated whether average performance on five books (i.e., Geometry, Basic Arithmetic, Units, Advanced arithmetic and Fractions) students worked through before processing problem sets of the algebra book, predicted their overall performance on the algebra book. This study did not evaluate whether specific fraction subtopics predict specific algebra subtopics and, if so, which ones.

References

Anderson, J. R., C. F. Boyle, and B. J. Reiser. 1985. "Intelligent Tutoring Systems." *Science* 228: 246–255. <https://doi.org/10.4324/9781315617572>.

Anderson, J. R., A. T. Corbett, K. R. Koedinger, and R. Pelletier. 1995. "Cognitive Tutors: Lessons Learned." *Journal of the Learning Sciences* 4, no. 2: 167–207.

Bailey, D. H., M. K. Hoard, L. Nugent, and D. C. Geary. 2012. "Competence With Fractions Predicts Gains in Mathematics Achievement." *Journal of Experimental Child Psychology* 113, no. 3: 447–455. <https://doi.org/10.1016/j.jecp.2012.06.004>.

Bailey, D. H., R. S. Siegler, and D. C. Geary. 2014. "Early Predictors of Middle School Fraction Knowledge." *Developmental Science* 17, no. 5: 775–785. <https://doi.org/10.1111/desc.12155>.

Barbieri, C. A., L. K. Young, K. J. Newton, and J. L. Booth. 2021. "Predicting Middle School Profiles of Algebra Performance Using Fraction Knowledge." *Child Development* 92, no. 5: 1984–2005.

Bardach, L., N. Hübner, B. Nagengast, U. Trautwein, and S. von Stumm. 2023. "Personality, Intelligence, and Academic Achievement: Charting Their Developmental Interplay." *Journal of Personality* 91, no. 6: 1326–1343. <https://doi.org/10.1111/jopy.12810>.

Booth, J. L., and K. J. Newton. 2012. "Fractions: Could They Really Be the Gatekeeper's Doorman?" *Contemporary Educational Psychology* 37, no. 4: 247–253. <https://doi.org/10.1016/j.cedpsych.2012.07.001>.

Borsboom, D., and A. O. Cramer. 2013. "Network Analysis: An Integrative Approach to the Structure of Psychopathology." *Annual Review of Clinical Psychology* 9: 91–121. <https://doi.org/10.1146/annur-ev-clinpsy-050212-185608>.

Borsboom, D., M. K. Deserno, M. Rhemtulla, et al. 2021. "Network Analysis of Multivariate Data in Psychological Science." *Nature Reviews Methods Primers* 1, no. 1: 58.

Boschloo, L., C. D. Van Borkulo, M. Rhemtulla, K. M. Keyes, D. Borsboom, and R. A. Schoevers. 2015. "The Network Structure of Symptoms of the Diagnostic and Statistical Manual of Mental Disorders." *PLoS One* 10, no. 9: 1–12. <https://doi.org/10.1371/journal.pone.0137621>.

Bower, C. A., K. S. Mix, L. Yuan, and L. B. Smith. 2022. "A Network Analysis of Children's Emerging Place-Value Concepts." *Psychological Science* 33, no. 7: 1112–1127. <https://doi.org/10.1177/09567976211070242>.

Corbett, A. T., K. Koedinger, and W. S. Hadley. 2001. "Cognitive Tutors: From the Research Classroom to All Classrooms." In *Technology Enhanced Learning*, 215–240. Routledge.

Costantini, G., S. Epskamp, D. Borsboom, et al. 2015. "State of the aRT Personality Research: A Tutorial on Network Analysis of Personality Data in R." *Journal of Research in Personality* 54: 13–29. <https://doi.org/10.1016/j.jrp.2014.07.003>.

Cramer, A. O., S. van der Sluis, A. Noordhof, et al. 2012. "Dimensions of Normal Personality as Networks in Search of Equilibrium: You Can't Like Parties if You Don't Like People." *European Journal of Personality* 26, no. 4: 414–431. <https://doi.org/10.1002/per.1866>.

Cramer, A. O., L. J. Waldorp, H. L. Van Der Maas, and D. Borsboom. 2010. "Comorbidity: A Network Perspective." *Behavioral and Brain Sciences* 33, no. 2–3: 137–150. <https://doi.org/10.1017/S0140525X09991567>.

Dalege, J., D. Borsboom, F. Van Harreveld, H. Van den Berg, M. Conner, and H. L. Van der Maas. 2016. "Toward a Formalized Account of Attitudes: The Causal Attitude Network (CAN) Model." *Psychological Review* 123, no. 1: 2–22. <https://doi.org/10.1037/a0039802>.

Epskamp, S., D. Borsboom, and E. I. Fried. 2018. "Estimating Psychological Networks and Their Accuracy: A Tutorial Paper." *Behavior Research Methods* 50, no. 1: 195–212. <https://doi.org/10.3758/s13428-017-0862-1>.

Epskamp, S., A. O. Cramer, L. J. Waldorp, V. D. Schmittmann, and D. Borsboom. 2012. "Qgraph: Network Visualizations of Relationships in Psychometric Data." *Journal of Statistical Software* 48, no. 4: 1–18. <https://doi.org/10.18637/jss.v048.i04>.

- Epskamp, S., and E. I. Fried. 2018. "A Tutorial on Regularized Partial Correlation Networks." *Psychological Methods* 23, no. 4: 617–634. <https://doi.org/10.1037/met0000167>.
- Fruchterman, T. M. J., and E. M. Reingold. 1991. "Graph Drawing by Force-Directed Placement." *Software: Practice and Experience* 21, no. 11: 1129–1164. https://doi.org/10.1007/978-3-319-64471-4_31.
- Hilz, A., K. Guill, J. Roloff, K. Aldrup, and O. Köller. 2023a. "The Relationship Between Individual Characteristics and Practice Behaviour Within an Adaptive Arithmetic Learning Program." *Journal of Computer Assisted Learning* 39, no. 3: 970–983. <https://doi.org/10.1111/jcal.12780>.
- Hilz, A., K. Guill, J. Roloff, D. Sommerhoff, and K. Aldrup. 2023b. "How to Continue? New Approaches to Investigating the Effects of Adaptive Math Learning Programs on Students' Performance, Self-Concept, and Anxiety." *Journal of Intelligence* 11, no. 6: 108. <https://doi.org/10.3390/jintelligence11060108>.
- Khazanchi, R., D. Di Mitri, and H. Drachsler. 2024. "The Effect of AI-Based Systems on Mathematics Achievement in Rural Context: A Quantitative Study." *Journal of Computer Assisted Learning* 41: e13098.
- Koedinger, K. R., P. F. Carvalho, R. Liu, and E. A. McLaughlin. 2023. "An Astonishing Regularity in Student Learning Rate." *Proceedings of the National Academy of Sciences of the United States of America* 120, no. 13: e2221311120. <https://doi.org/10.1073/pnas>.
- Libertus, M. E., L. Feigenson, and J. Halberda. 2011. "Preschool Acuity of the Approximate Number System Correlates With School Math Ability." *Developmental Science* 14, no. 6: 1292–1300.
- Malmberg, J., M. Saqr, H. Järvenoja, and S. Järvelä. 2022. "How the Monitoring Events of Individual Students Are Associated With Phases of Regulation—A Network Analysis Approach." *Journal of Learning Analytics* 9, no. 1: 77–92. <https://doi.org/10.18608/jla.2022.7429>.
- Mavrikis, M., N. Rummel, M. Wiedmann, K. Loibl, and W. Holmes. 2022. "Combining Exploratory Learning With Structured Practice Educational Technologies to Foster Both Conceptual and Procedural Fractions Knowledge." *Educational Technology Research and Development* 70, no. 3: 691–712. <https://doi.org/10.1007/s11423-022-10104-0>.
- McMullen, J., and J. Van Hoof. 2020. "The Role of Rational Number Density Knowledge in Mathematical Development." *Learning and Instruction* 65: 101228.
- Meeter, M. 2021. "Primary School Mathematics During Covid-19: No Evidence of Learning Gaps in Adaptive Practicing Results." *Trends in Neuroscience and Education* 25: 100163. <https://doi.org/10.1016/j.tine.2021.100163>.
- Meyer, J., T. Jansen, N. Hübner, and O. Lüdtke. 2023. "Disentangling the Association Between the Big Five Personality Traits and Student Achievement: Meta-Analytic Evidence on the Role of Domain Specificity and Achievement Measures." *Educational Psychology Review* 35, no. 1: 1–34. <https://doi.org/10.1007/s10648-023-09736-2>.
- Mou, Y., Y. Li, M. K. Hoard, et al. 2016. "Developmental Foundations of Children's Fraction Magnitude Knowledge." *Cognitive Development* 39: 141–153. <https://doi.org/10.1016/j.cogdev.2016.05.002>.
- Murnane, R. J., J. B. Willett, and F. Levy. 1995. "The Growing Importance of Cognitive Skills in Wage Determination." *Review of Economics and Statistics* 77, no. 2: 251–266.
- National Mathematics Advisory Panel. 2008. *Foundations for Success: The Final Report of the National Mathematics Advisory Panel*. US Department of Education.
- Park, S., and A. G. Esposito. 2024. "Rational Number Representation, Math Anxiety, and Algebra Performance in College Students." *Cognitive Development* 69: 101417.
- R Core Team. 2013. *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing. <http://www.r-project.org/>.
- Rau, M. A., V. Alevén, and N. Rummel. 2009. "Intelligent Tutoring Systems With Multiple Representations and Self-Explanation Prompts Support Learning of Fractions." *Frontiers in Artificial Intelligence and Applications* 200, no. 1: 441–448. <https://doi.org/10.3233/978-1-60750-028-5-441>.
- Ritter, S., J. R. Anderson, K. R. Koedinger, and A. Corbett. 2007. "Cognitive Tutor: Applied Research in Mathematics Education." *Psychonomic Bulletin & Review* 14, no. 2: 249–255.
- Rittle-Johnson, B., and K. Koedinger. 2009. "Iterating Between Lessons on Concepts and Procedures Can Improve Mathematics Knowledge." *British Journal of Educational Psychology* 79, no. 3: 483–500. <https://doi.org/10.1348/000709908X398106>.
- Rittle-Johnson, B., and K. R. Koedinger. 2005. "Designing Knowledge Scaffolds to Support Mathematical Problem Solving." *Cognition and Instruction* 23, no. 3: 313–349. https://doi.org/10.1207/s1532690xc12303_1.
- Rittle-Johnson, B., R. S. Siegler, and M. W. Alibali. 2001. "Developing Conceptual Understanding and Procedural Skill in Mathematics: An Iterative Process." *Journal of Educational Psychology* 93, no. 2: 346–362.
- Siegler, R. S., and D. W. Braithwaite. 2017. "Numerical Development." *Annual Review of Psychology* 68: 187–213. <https://doi.org/10.1146/annurev-psych-010416-044101>.
- Siegler, R. S., G. J. Duncan, P. E. Davis-Kean, et al. 2012. "Early Predictors of High School Mathematics Achievement." *Psychological Science* 23, no. 7: 691–697. <https://doi.org/10.1177/0956797612440101>.
- Siegler, R. S., and H. Lortie-Forgues. 2014. "An Integrative Theory of Numerical Development." *Child Development Perspectives* 8: 144–150.
- Siegler, R. S., C. A. Thompson, and M. Schneider. 2011. "An Integrated Theory of Whole Number and Fractions Development." *Cognitive Psychology* 62, no. 4: 273–296. <https://doi.org/10.1016/j.cogpsych.2011.03.001>.
- Spitzer, M. W. H. 2022. "Just Do It! Study Time Increases Mathematical Achievement Scores for Grade 4-10 Students in a Large Longitudinal Cross-Country Study." *European Journal of Psychology of Education* 37: 39–53. <https://doi.org/10.1007/s10212-021-00546-0>.
- Spitzer, M. W. H., L. Bardach, Y. Strittmatter, J. Meyer, and K. Moeller. 2024. "Evaluating the Content Structure of Intelligent Tutor Systems—A Psychological Network Analysis." *Computers and Education Open* 7: 100198.
- Spitzer, M. W. H., L. Bardach, Y. Strittmatter, and K. Moeller. 2025. "Usage and Performance Declines in a Classroom-Integrated Digital Learning Software Over the Course of an Academic Year." In *Proceedings of the 15th International Learning Analytics and Knowledge Conference*, 957–962. Association for Computing Machinery.
- Spitzer, M. W. H., and K. Moeller. 2022. "Predicting Fraction and Algebra Achievements Online: A Large-Scale Longitudinal Study Using Data From an Online Learning Environment." *Journal of Computer Assisted Learning* 38, no. 6: 1797–1806. <https://doi.org/10.31234/osf.io/rw6b9>.
- Spitzer, M. W. H., K. Moeller, and S. Musslick. 2023. "Assignment Strategies Modulate Students' Academic Performance in an Online Learning Environment During the First and Second Covid-19 Related School Closures." *PLoS One* 18, no. 5: e0284868.
- Spitzer, M. W. H., M. Ruiz-Garcia, and K. Moeller. 2025. "Basic Mathematical Skills and Fraction Understanding Predict Percentage Understanding: Evidence From an Intelligent Tutoring System." *British Journal of Educational Technology* 56, no. 3: 1122–1147.
- Spitzer, M. W. H., M. Ruiz-Garcia, Y. Strittmatter, E. Richter, R. Gutsfeld, and K. Moeller. 2024. "Achievements in Arithmetic and Measurement Units Predict Fraction Understanding in an Additive and Linear Manner." *Cognitive Development* 72: 101517.
- Stapel, M., Z. Zheng, and N. Pinkwart. 2016. "An Ensemble Method to Predict Student Performance in an Online Math Learning Environment."

In *Proceedings of the 9th International Conference on Educational Data Mining*, 231–238. Association for Computing Machinery.

Tang, X., K. A. Renninger, S. E. Hidi, K. Murayama, J. Lavonen, and K. Salmela-Aro. 2022. “The Differences and Similarities Between Curiosity and Interest: Meta-Analysis and Network Analyses.” *Learning and Instruction* 80: 101628. <https://doi.org/10.1016/j.learninstruc.2022.101628>.

Tomasik, M. J., L. A. Helbling, and U. Moser. 2020. “Educational Gains of In-Person vs. Distance Learning in Primary and Secondary Schools: A Natural Experiment During the COVID-19 Pandemic School Closures in Switzerland.” *International Journal of Psychology* 56, no. 4: 566–576. <https://doi.org/10.1002/ijop.12728>.

Torbeyns, J., M. Schneider, Z. Xin, and R. S. Siegler. 2015. “Bridging the Gap: Fraction Understanding Is Central to Mathematics Achievement in Students From Three Different Continents.” *Learning and Instruction* 37: 5–13.

van Hoogmoed, A. H., P. Adriaanse, M. Vermeiden, and R. Weggemans. 2024. “Combining Cognitive and Affective Factors Related to Mathematical Achievement in 4th Graders: A Psychological Network Analysis Study.” *Trends in Neuroscience and Education* 37: 100241.

VanLehn, K. 2011. “The Relative Effectiveness of Human Tutoring, Intelligent Tutoring Systems, and Other Tutoring Systems.” *Educational Psychologist* 46, no. 4: 197–221.

Whalen, K. A., A. Renkl, A. Eitel, and I. Glogger-Frey. 2024. “Digital Re-Attributional Feedback in High School Mathematics Education and Its Effect on Motivation and Achievement.” *Journal of Computer Assisted Learning* 40, no. 2: 478–493.

Supporting Information

Additional supporting information can be found online in the Supporting Information section.