Evaluation of the Noise Immunity of the MIAM Communication System

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Abstract: The article proposes a modification of the known invariant amplitude modulation that transmits the values of information elements by the ratio of the lengths of the signal vectors lying on a straight line passing through the origin of the coordinate system of the signal space [1]. Modification of this modulation makes it possible to use signals whose vector ends lie on a straight line that does not necessarily pass through the origin of the signal space coordinate system. This gives the opportunity to use in a greater variety of signals, not just signals of similar shape as in the well-known invariant of amplitude modulation that can be useful to enhance immunity against a specific type of interference and to secrecy of messages transmitted. The article contains an assessment of the noise immunity of a communication system with modified invariant amplitude modulation to white noise and a description of its structural scheme.

1 INTRODUCTION

Currently, it is established [1] that the effect on the signals of the communication channel can be reflected by the corresponding group of transformations. For example, the change in signals in linear channels is described by an affine transformation group, a subgroup of which is the group of orthogonal transformations. The latter corresponds to the case when the channel has a flat amplitude-frequency and linear phase-frequency characteristics. The effect of additive interference is displayed by a group of shifts of the ends of the signal vectors in the direction of the interference vector.

The proof of the possibility of describing a channel by a transformation group opens up a method for undistorted message transmission by using group invariants - special relations between signal parameters that remain unchanged despite changes in the signals themselves by the channel [1].

An exception is the distortion of signals by white noise, which can only be absolutely invariant by applying signals with infinitely high energy.

It can be shown that the «classical» amplitude, frequency, and relative phase modulations also use invariants of the simplest orthogonal transformation group, which preserves the length of the signal vectors and the angle difference between them.

To date, the basic properties of the invariant amplitude modulation, which uses the basic invariant of the affine transformation group, describing the entire class of linear communication channels with arbitrary frequency characteristics, have been partially studied.

At the same time, such an invariant, called in mathematics "the ratio of three points" [2], in relation to communication problems, has so far been formulated as a channel preserving the ratio of the lengths of signal vectors lying on a single straight line passing through the origin of the coordinate system of the signal space. However, the " ratio of three points "(the ratio of the lengths of segments lying on the same line and defined by these points preserved by an affine transformation) is also valid for the General case when the line does not pass through the origin [2]. The synthesis of modified invariant amplitude modulation (MIAM) and demodulation algorithms for this General case is described below.

2 SYNTHESIS OF MODIFIED INVARIANT AMPLITUDE MODULATION AND DEMODULATION

Below, in order to provide greater clarity of the synthesis procedure, a two-dimensional signal space is used, the coordinate axes of which correspond to some orthonormal basis functions of time. For example, the Kotelnikov functions $\varphi_1(t)$ and $\varphi_2(t)$, which differ in the time shift that ensures their orthogonality. In this case, the signals can be represented by two time counts, the values of which set the coordinates of the ends of the signal vectors in a two-dimensional signal space (Figure 1).

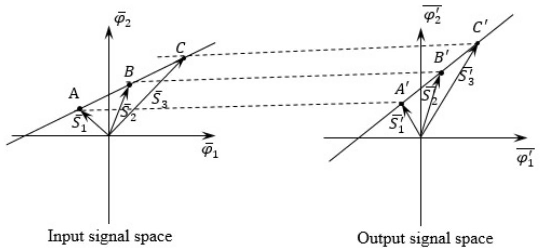

Figure 1: The scheme is an affine transformation of the input signals at the output signals for the modified invariant amplitude modulation.

Figure 1 on the left shows a straight line that occupies a general position and does not pass through the origin of the coordinate systems of the signal space. Points A, B, and C define the ends of the three input vectors $\bar{S}_1$, $\bar{S}_2$ and $\bar{S}_3$.

As mentioned above, the transformation of input signals into output signals is described by an affine transformation group. In the Figure 1 the dotted lines represent the scheme of affine transformation of the ends of the input signal vectors into the ends of the corresponding output signal vectors $\bar{S}'_1$, $\bar{S}'_2$ and $\bar{S}'_3$. One of the possible forms of writing the affine transformation invariant in the form of the "three-point relation" in this example has the following form (1) and (2).

$$J = \frac{BC}{AB} = \frac{B'C'}{A'B'} = \frac{|\bar{S}_3 - \bar{S}_2|}{|\bar{S}_2 - \bar{S}_1|} = \frac{|\bar{S}'_3 - \bar{S}'_2|}{|\bar{S}'_2 - \bar{S}'_1|}. \quad (1)$$

Such an invariant entry is also possible

$$J = \frac{AC}{AB} = \frac{A'C'}{A'B'} = \frac{|\bar{S}_3 - \bar{S}_1|}{|\bar{S}_2 - \bar{S}_1|} = \frac{|\bar{S}'_3 - \bar{S}'_1|}{|\bar{S}'_2 - \bar{S}'_1|}. \quad (2)$$

We assume that in (1) $S_1(t)$ and $S_2(t)$ perform the role of so-called "reference signals", and $S_3(t)$ – informational $S_i(t)$. In this case, the modified invariant amplitude modulation (MIAM) algorithm can be obtained from (1) - the algorithm modulation A (3):

$$\bar{S}_i = J_i\left(\bar{S}_2 - \bar{S}_1\right) + \bar{S}_2, \quad (3)$$

where:

J_i - value of the transmitted information element;

$\bar{S}_i$ - the vector of the information signal $S_i(t)$, which together with the reference signals $S_1(t)$ and $S_2(t)$ transmits the value of the information element J_i;

i - number of the time interval during which the value of the information element J_i is transmitted.

From (1) and Figure 1 follows the algorithm demodulation A (4):

$$\hat{J}_i = \frac{\left|\hat{\bar{S}}'_i - \hat{\bar{S}}'_2\right|}{\left|\hat{\bar{S}}'_2 - \hat{\bar{S}}'_1\right|} \text{ -}. \quad (4)$$

Here, the ^ sign indicates the estimates of the value J_i at the output of the demodulator and the vectors $\hat{\bar{S}}'_1$, $\hat{\bar{S}}'_2$ and $\hat{\bar{S}}'_3$ at the input of the demodulator.

Expression (2) gives other equivalent modulation and demodulation algorithms - the algorithm modulation B (5) and the algorithm demodulation B (6):

$$\bar{S}_i = J_i(\bar{S}_2 - \bar{S}_1) + \bar{S}_1, \quad (5)$$

$$\hat{J}_i = \frac{\left|\hat{\bar{S}}'_i - \hat{\bar{S}}'_1\right|}{\left|\hat{\bar{S}}'_2 - \hat{\bar{S}}'_1\right|}. \quad (6)$$

3 ESTIMATION OF NOISE IMMUNITY OF MODIFIED INVARIANT AMPLITUDE MODULATION TO WHITE NOISE

Let us denote the modules of the difference vectors for brevity $|\hat{\bar{S}}'_i - \hat{\bar{S}}'_1|$, $|\hat{\bar{S}}'_i - \hat{\bar{S}}'_2|$ and $|\hat{\bar{S}}'_2 - \hat{\bar{S}}'_1|$ as $\left|\Delta\hat{\bar{S}}'_{i,1}\right|$, $\left|\Delta\hat{\bar{S}}'_{i,2}\right|$ и $\left|\Delta\hat{\bar{S}}'_{2,1}\right|$, respectively. Then the demodulation algorithms A and B can be written as follows (7) and (8):

$$\hat{J}_i = \frac{|\Delta\hat{\bar{S}}'_{i,2}|}{|\Delta\hat{\bar{S}}'_{2,1}|}; \qquad (7)$$

$$\hat{J}_i = \frac{|\Delta\hat{\bar{S}}'_{i,1}|}{|\Delta\hat{\bar{S}}'_{2,1}|}. \qquad (8)$$

Taking into account the influence of white noise $n(t)$ on the transmitted signals $S_i(t)$, $S_2(t)$, and $S_1(t)$ for estimating the lengths of difference vectors $\Delta\bar{S}'_{i,1}$, $\Delta\bar{S}'_{i,2}$ and $\Delta\bar{S}'_{2,1}$ can be written (9), (10), (11)

$$|\Delta\hat{\bar{S}}'_{i,1}| = |\Delta\bar{S}'_{i,1}| + n_i - n_1; \qquad (9)$$

$$|\Delta\hat{\bar{S}}'_{i,2}| = |\Delta\bar{S}'_{i,2}| + n_i - n_2; \qquad (10)$$

$$|\Delta\hat{\bar{S}}'_{2,1}| = |\Delta\bar{S}'_{2,1}| + n_2 - n_1, \qquad (11)$$

where n_i, n_2, n_1 - are the values of the projection lengths of the vectors of realizations of white noise interference on the direction set by the line passing through the points A', B', C' (Figure 1), and affecting the transmitted signals $Si(t)$, $S_2(t)$, and $S_1(t)$, respectively.

It is known that the projections of the white noise realization vector in any orthonormal basis are Gaussian random variables with $\sigma^2_{w.n.}$ and zero expectation [3].

With this in mind (9), (10), (11) they are Gaussian random variables with mathematical expectations $|\Delta\bar{S}'_{i,1}|, |\Delta\bar{S}'_{i,2}|$ и $|\Delta\bar{S}'_{2,1}|$ and dispersions $2\sigma^2_{w.n.}$.

Values $\hat{J}_i$ in algorithms (7) and (8) are functionally transformed random variables. As is known [4], for a functionally transformed random variable in the form $y = \frac{x_2}{x_1}$, the following inequality holds (12):

$$\omega(y) = \int_{-\infty}^{\infty} \omega(x_1, x_2)|x_1|dx_1 = \int_{-\infty}^{\infty} \omega(x_1, yx_1)|x_1|dx_1, \qquad (12)$$

where $\omega(x_1, x_2 = yx_1)$ - two-dimensional law of probability distribution x_1 and x_2.

In our case, taking into account the independence of random variables in the numerators and denominators of formulas (7) and (8), we have:

$$\omega\left(\left|\Delta\hat{\bar{S}}'_{i,2}\right|, \left|\Delta\hat{\bar{S}}'_{2,1}\right|\right) = \omega\left(\left|\Delta\hat{\bar{S}}'_{i,2}\right|\right)\omega\left(\left|\Delta\hat{\bar{S}}'_{2,1}\right|\right)$$

$$\omega\left(\left|\Delta\hat{\bar{S}}'_{i,1}\right|, \left|\Delta\hat{\bar{S}}'_{2,1}\right|\right) = \omega\left(\left|\Delta\hat{\bar{S}}'_{i,1}\right|\right)\omega\left(\left|\Delta\hat{\bar{S}}'_{2,1}\right|\right)$$

In accordance with (12), we obtain the following expression for the conditional probability density of estimates of the values of information elements $\hat{J}_i$ at the output of the demodulator for algorithm A (13):

$$\omega\left(\hat{J}_i / J_i\right) = \frac{1}{4\pi\sigma^2_{w.n.}} \int_{-\infty}^{\infty} e^{-a} \left|\Delta\hat{\bar{S}}'_{2,1}\right| d\left|\Delta\hat{\bar{S}}'_{2,1}\right|, \qquad (13)$$

where $a = \dfrac{\left(\left|\Delta\hat{\bar{S}}'_{2,1}\right| - \left|\Delta\bar{S}'_{2,1}\right|\right)^2 + \left[\left(\hat{J}_i - J_i\right)\left|\Delta\bar{S}'_{2,1}\right|\right]^2}{4\sigma^2_{w.n.}}$

For example, graphs were calculated in the MATLAB environment $\omega(\hat{J}_i / J_i)$ (Figures 2 - 5) when using reference signals with the length of the difference vector $\left|\Delta\bar{S}'_{2,1}\right|$=1 and $\left|\Delta\bar{S}'_{2,1}\right|$=2 and values of transmitted information elements J_i = 1, 2, 3, 4, 5, 6 for interference power $\sigma^2_{w.n.}$=0,1 and $\sigma^2_{w.n.}$=0,2 [5].

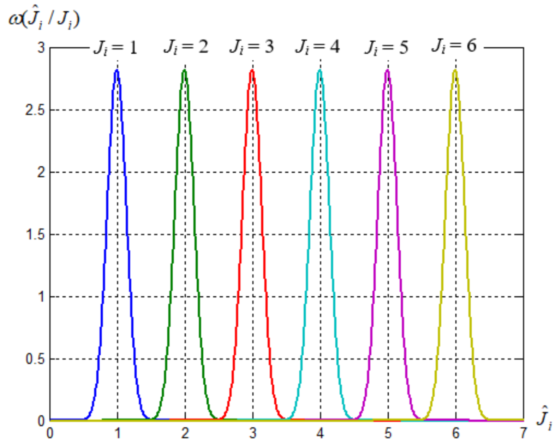

Figure 2: Graphics $\omega(\hat{J}_i / J_i)$ for $\left|\Delta\bar{S}'_{2,1}\right|$=1, $\sigma^2_{w.n.}$=0,1.

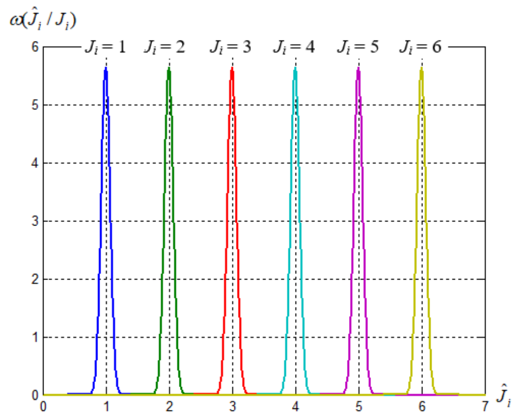

Figure 3: Graphics $\omega(\hat{J}_i / J_i)$ for $|\Delta\bar{S}'_{2,1}|$=2, $\sigma^2_{w.n.}$=0,1.

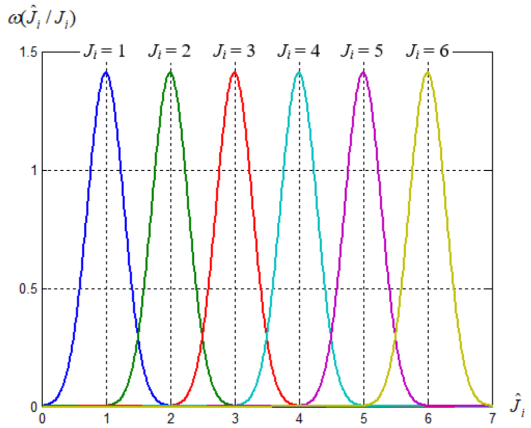

Figure 4: Graphics $\omega(\hat{J}_i / J_i)$ for $|\Delta\bar{S}'_{2,1}|$=1, $\sigma^2_{w.n.}$=0,2.

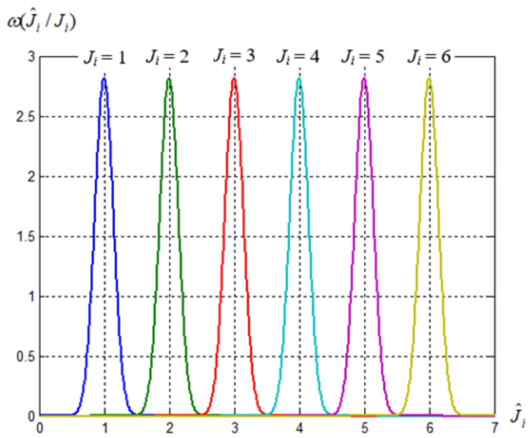

Figure 5: Graphics $\omega(\hat{J}_i / J_i)$ for $|\Delta\bar{S}'_{2,1}|$=2, $\sigma^2_{w.n.}$=0,2.

As follows from these graphs, an increase in the length of the vector of the difference between the vectors of two reference signals leads to a decrease in the variance of estimates of the values of information elements at the output of the demodulator. Therefore, by choosing the required length of the difference vector, it can be provided the necessary minimization of the area of mutual overlap of graphs $\omega(\hat{J}_i / J_i)$, that is, the required fidelity of the J_i transmission.

3 CONCLUSIONS

The proposed communication system with modified invariant modulation is a further generalization of the known system with invariant amplitude modulation [1].

The advantage of this communication system is that there is no need to ensure the similarity of the forms of the signals used, as is the case with the prototype. The line of location of the ends of the vectors of transmitted signals can occupy an arbitrary position. This circumstance can be used to increase the degree of secrecy of transmitted messages, similar to the well-known method of carrier frequency tuning.

In addition, when changing the sequence of transmission of reference signals, taking into account the lengths of their vectors, it becomes possible to transmit a sign of positivity or negativity of the values of information elements. For example, a positive sign may be that the first time reference signal has a shorter vector length compared to the vector length of the second time reference signal. The negative sign is transmitted in the reverse order of the reference signals [5].

Thus, the proposed modified invariant modulation allows to double the volume of the alphabet of information elements compared to the prototype.

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