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High-order SBFEM solution of the Reynolds equation

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A semi-analytical solution of the Reynolds equation for hydrodynamic bearings in rotordynamic simulations is investigated, which is based on the Scaled Boundary Finite Element Method (SBFEM). The numerical efficiency of this approach is compared to the Finite Element Method (FEM), considering linear as well as higher-order shape functions. It is observed that the SBFEM requires significantly less computational time than the FEM, especially with respect to high-order formulations.

1 Introduction

The numerical effort of transient rotordynamic simulations with hydrodynamic bearings is usually dominated by the computation of the bearing forces, which is necessary in every time step and requires a solution of the Reynolds equation. Multiple strategies for solving this equation exist, including numerical methods, such as the Finite Element Method (FEM) or the Finite Volume Method (FVM), as well as approximations based on analytical models or look-up tables. Since numerical solutions are able to satisfy all types of boundary conditions (BCs), they are often preferred, although this leads to a time-expensive overall simulation.

In [1], a semi-analytical solution of the Reynolds equation by means of the Scaled Boundary Finite Element Method (SBFEM) [2] is developed and compared to the FEM. It is observed that the relative efficiency of the two methods depends on the slenderness ratio of the bearing (length l divided by diameter d) and whether the oil supply groove is neglected, simplified, or accurately modeled. However, the investigation in [1] is limited to linear shape functions, and the computational time is only measured for the eigenvalue problems and linear equation systems. The study at hand, in contrast, includes higher-order shape functions and considers the total computational time, which still depends substantially on the eigenvalue problems and linear equation systems, but also on the integration of the element matrices and the assembly.

2 Comparison of the SBFEM to an FEM solution regarding numerical efficiency

The numerical efficiency of the SBFEM is compared to the FEM for linear and higher-order shape functions. To this end, the computational time per solution of the Reynolds equation is analyzed, while the node number and the polynomial degree of the shape functions are varied. Since the SBFEM solution requires a negligence of the shaft tilting [1], all simulations in this study are conducted without this effect. In numerical solutions, such as the FEM, this assumption is not mandatory, but in cases where this simplification is tolerated, the node number can be reduced. This is because without shaft tilting, the pressure fields are symmetric, which means that a symmetric BC can be used. As a result, only one half of the bearing needs to be discretized. Thus, analogously to [1], the SBFEM is compared to an FEM solution that exploits this numerical advantage. Moreover, a second FEM model without a symmetric BC (i.e., with a fully discretized bearing) is investigated, allowing to also analyze to what extent the numerical effort of the FEM is affected by whether shaft tilting is considered or neglected.

The integration of the element matrices is conducted numerically using Gauss-Lobatto-Legendre quadrature for all shape functions with a polynomial degree of $n \geq 2$ and analytically for $n = 1$ (linear elements). For the solution of the linear equation systems in the SBFEM and the FEM as well as the eigenvalue problems in the SBFEM, standard Matlab routines are employed. The corresponding algorithms, which depend on the respective matrix properties, are specified in [1]. An evaluation of the efficiency requires that not only the computational time but also the accuracy of the solutions is investigated. To this end, every computed pressure field is compared to a much finer discretized FEM solution, leading to a relative error as defined in [1]. The relative shaft eccentricity, which influences the accuracy of all investigated solutions to a certain extent, is set to $\varepsilon = 0.5$ as an exemplary value. Moreover, the slenderness ratio of the investigated bearing is chosen as $l/d = 1$. All simulations are conducted in Matlab R2019a on a desktop PC (Intel(R) Core(TM) i7-8700 CPU, 3.2 GHz; 64 GB RAM). Since the scope of content of this paper must be limited, only a case without oil supply BCs is investigated.

In Figure 1, the resulting computational times (horizontal axis) and logarithmic relative errors (vertical axis) are depicted. As expected, the FEM model with a symmetric BC (dashed lines) requires significantly less time (by a factor of $0.28 \ldots 0.65$) than the model without a symmetric BC (dotted lines). Both FEM models have in common that for logarithmic errors below -2.3, the numerical effort decreases as the polynomial degree is raised from $n = 1$ (blue) to $n = 2$ (red) and further

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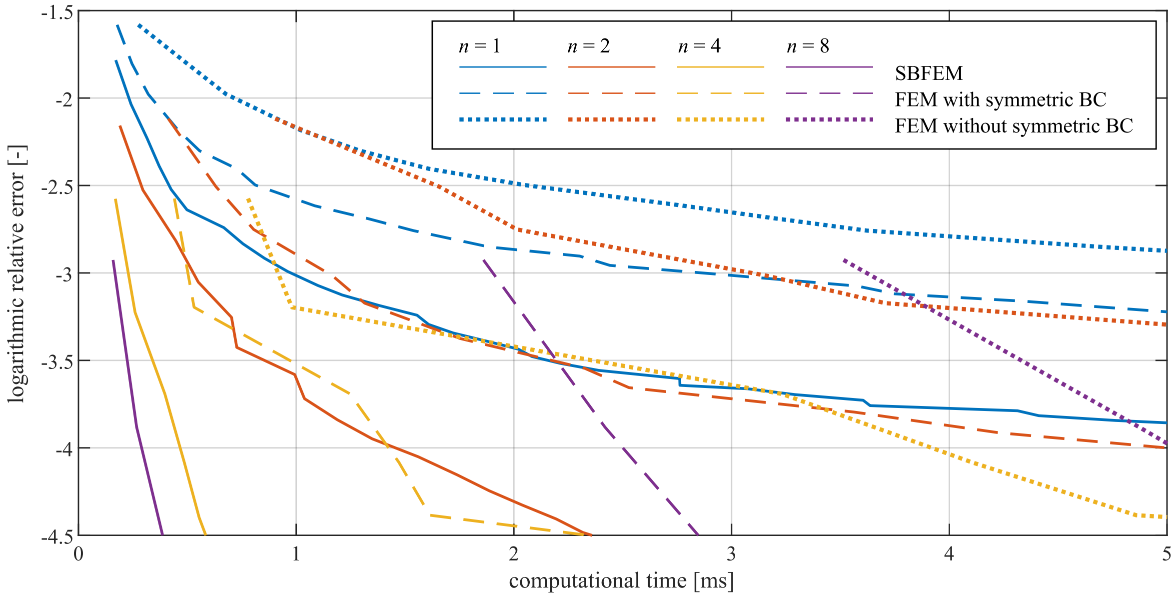

Fig. 1: Comparison of the error and the computational time of the SBFEM (solid lines) and two FEM models (dashed and dotted lines) for different node numbers (the node numbers vary along the line paths) and for shape functions of different polynomial degrees n (colors).

to $n = 4$ (yellow), although an increase to $n = 8$ (purple) leads to the opposite effect (which will be discussed in the next paragraph), as far as the investigated range of node numbers is concerned. The SBFEM (solid lines) consistently becomes more efficient every time n is incremented, including $n = 8$. It is observed that the SBFEM requires less computational time than the FEM, especially if a small error is desired. A comparison of the most efficient SBFEM and FEM discretizations for each accuracy shows that the SBFEM is by a factor between 0.16 and 0.73 less time-expensive than the FEM solution with a symmetric BC.

The notion that high-order shape functions are more advantageous in the SBFEM than in the FEM is known from other SBFEM applications and is already discussed in the literature [3, 4]. Incrementing the polynomial degree leads to a faster convergence, but as far as the FEM is concerned, the resulting numerical advantages are at some point outweighed by the increasing effort for the integration of the element matrices and the growing bandwidth of the system matrix. The SBFEM, in contrast, is hardly influenced by these negative aspects since the elements are of lower dimensionality and because the matrix bandwidth does not affect eigenvalue problems as it affects linear equation systems.

3 Conclusion and outlook

Given that shaft tilting may be neglected, a symmetric BC allows reducing the computational time of the FEM by a factor between 0.28 and 0.65. The SBFEM solution, in which this simplification is mandatory, provides an even more efficient alternative for this case, as it requires only 0.16 to 0.73 times the numerical effort of the FEM solution with a symmetric BC. It should be noted that no oil supply BCs are considered in the study at hand, allowing to conduct the SBFEM solution with a single superelement [1]. For a model with a geometrically accurate oil supply groove, two superelements are required, increasing the numerical effort of the SBFEM [1]. This case will be investigated for high-order shape functions in a later study. Moreover, the SBFEM solution will be combined with a cavitation model and incorporated into rotordynamic simulations of complex technical systems. Since the efficiency of the SBFEM solution has only been compared to the FEM, the FVM will be considered in the future as well.

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